An Empirical Assessment of the Double Exponential Jump-Diffusion Process*

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ABSTRACT

The double exponential jump-diffusion (DEJD) model, recently proposed by Kou (2002) and Ramezani & Zeng (1998), generates a highly skewed and leptokurtic distribution and is capable of matching key features of stock and index returns. Moreover, DEJD leads to tractable pricing formulas for exotic and path dependent options (Kou & Wang 2004). Accordingly, the Double Exponential representation has gained wide acceptance. However, estimation and empirical assessment of this model has received little attention to date. The primary objective of this paper is to fill this gap. We use daily returns for NYSE and NASDAQ firms and daily and monthly returns for the S&P-500 and the NASDAQ indexes, in conjunction with maximum likelihood estimation to fit the DEJD model. We utilize the BIC criterion to assess the performance of DEJD relative to a number of alternatives. For individual stocks, based on data spanning the period 10/96–12/98, we find that relative to the Log-normal Jump-Diffusion (LJD), the DEJD provides a better fit for only 11% of the sampled firms. For these firms and the indexes, we compare DEJD to six popular versions of the ARCH specification using data for the period 1/1999 through 12/2003. The performance of DEJD for individual stocks is strong for this period: DEJD performs better than LJD and ARCH for majority of cases. For indexes the ARCH alternatives dominate, but the DEJD provides a better fit than LJD. Overall the empirical evidence in support of DEJD is mixed.

Keywords: Asset Price Processes, Double Exponential Jump-Diffusion, Pareto-Beta Jump Diffusion, Leptokurtic Distributions, Volatility Smile and Smirk, MLE

JEL Classification: C32, C52, G12, G13

1 Introduction

Nearly three decades has passed since the seminal papers by Robert Merton (1976a, 1976b), suggested that asset price dynamics may be modeled as Jump-Diffusion (JD) processes and provided the foundations to value contingent claims under this specification. Building on the works of Press (1967), Merton posited that the returns processes consists of three components, a linear drift, a Brownian motion representing “normal” price variations, and a compound Poisson process that accounts for “abnormal” change in prices (jumps) generated by “news” arrivals. Upon the arrival of news, jump magnitudes are determined by sampling from an independent and identically distributed (IID) random variable. For the purpose of pricing options, Merton further assumed that the jump magnitude are log-normally distributed (LJD hereafter) . This special case renders estimation and hypothesis testing tractable and has become the most important representation of the Jump-Diffusion process.

Almost every aspect of modern finance, from valuations and portfolio choice to option pricing and corporate finance, as well as the ever expanding field of financial econometrics, critically depend upon the form of the probability distribution describing the dynamics of security prices. Although the Geometric Brownian Motions (GBM) had served as a convenient and tractable framework, as the empirical evidence against GBM accumulated (see Sundaresan (2000)), Merton’s JD representation gained wide acceptance, primarily because it was shown to be more consistent with the empirical return distributions (higher peak and excess kurtosis and skewness). Moreover, empirical evidence indicates that option pricing formulas based on JD representation exhibit less bias –i.e. better explain the “smile” and “skew” across both moneyness and maturity (see Bakshi, Cao & Chen (1997) and the survey by Garcia, Ghysels & Renault (2004)). Chernov, Gallant, Ghysels & Tauchen (2003), Eraker, Johannes & Polson (2003), and the papers in Aït-Sahalia & Hansen (2004) provide a complete survey of the most important developments in the JD literature, focusing on the econometric issues. Bates (2003*a*) provides an assessment of the progress in this area and highlights some of the remaining challenges.[1]

The JD class of representations are composed of three building blocks: a drift, a diffusion, and a jump component. Generalizations occur by assuming different theoretical structure for each block (e.g., stochastic volatility and mean reversion). Numerous variations have been proposed to enhance the jump specification, including different distributional assumptions for the jump magnitude, time varying jump intensity, and correlated jump magnitudes, to name a few. For example, Naik (1993) proposed a model in which price volatility jump from one regime to another. Andersen & Andreasen (2000) combined a deterministic volatility structure with log-normally distributed Poisson jumps. Bakshi & Cao (2004) and Eraker et al. (2003) considered models in which both stock price and volatility jump.[2]

In the end, a large number of continuous time models emerge, primarily by choosing different specifications for the basic building blocks. Each specific formulations can be shown to be a special case of the Affine Jump Diffusion (AJD) framework of Duffie, Pan & Singleton (2000). Huang & Wu (2004) provide a systematic catalog of the AJD alternatives and their properties. The popularity of AJD framework is due to its modeling flexibility that captures important features of financial processes, and its technical tractability in deriving standard and extended transforms for option and bond pricing, as well as econometric estimation.

The Double Exponential Jump Diffusion (DEJD) is a special case of the AJD family but it has desirable properties for both pricing exotic options and econometric estimation. In its most popular form, the jump-diffusion model has a single jump component that captures the impact of news on security prices. News that cause upward jump in prices –"good news"– and news that cause downward jump in prices –"bad news"– are not distinguished by their intensity or distributional characteristics. This potential limitation of the simple jump-diffusion framework has lead to two alternative specifications.[3] Under Kou's (2002) DEJD specification, a single Poisson process with fixed intensity generates the jumps in prices, but the jump magnitudes are drawn from two independent exponential distributions. Ramezani & Zeng (1998) independently propose the Pareto-Beta Jump-Diffusion (PBJD), assuming that good and bad news are generated by two independent Poisson processes and jump magnitudes are drawn from the Pareto and Beta distributions. Below we show that the two models are closely related in that the parameters of one model can be exactly recovered from the other.[4]

The DEJD model has gained popularity primarily because its distribution is asymmetric and leptokurtic. Furthermore, as Kou (2002), Sepp (2004) and Kou & Wang (2004) have shown, the DEJD model leads to nearly analytical solutions to many option pricing problems, including certain exotic and path dependent options. This is a significant advantage as most of the existing methods for pricing options under the jump-diffusion processes are confined to plain vanilla European options. Because of these features, the DEJD has been used to model the price process for stocks indexes, as well as pricing credit risk instruments tied to individual stocks.[5] However, estimation and empirical assessment of this model has received little attention to date.[6] In practice, most studies have arbitrarily selected "reasonable" parameter values, because as Huang & Huang (2003) observe "no study has estimated the parameters for this model". The primary objective of this paper is to remedy this deficiency by providing an empirical assessment of the DEJD specification.

Using daily data for a large sample of firms and daily and monthly data for the S&P-500 and the NASDAQ composite, we undertake a comprehensive empirical evaluation of DEJD. We utilize the BIC criterion to assess the performance of DEJD relative to a number of alternatives. For individual stocks, we find that relative to the LJD, the DEJD provides a better fit for only 11% of the firms. For these firms and the indexes, we compare DEJD to six popular versions of the Autoregressive Conditional Heteroskedasticity (ARCH) specification. The performance of DEJD for individual stocks remains weak. For indexes the ARCH alternatives dominate, but the DEJD provides a better fit than LJD. Overall the empirical evidence in support of DEJD is mixed.

2 The Model

In this section we first present the Pareto-Beta Jump-Diffusion (PBJD) model, which assumes that good and bad news are generated by two independent Poisson processes and jump magnitudes are drawn from the Pareto and Beta distributions (Ramezani & Zeng 1998). We then identify the conditions under which PBJD reduces to DEJD specification, which has a single Poisson process generating news and two independent exponential distributions generating the up and down jump magnitudes. These models represent simple extensions to Merton's (1976a) JD model. While the motivation for the DEJD is to price exotic options and explain the "volatility smile," PBJD is put forth to statistically test the hypothetical distinction between good and bad news and their associated intensities.

There are several economic justifications for making a distinction between good and bad news. At a microeconomic level, Milgrom (1981) has formalized the notion of good and bad news and shown that such distinction plays an important role in rational expectation models that are the foundation of information economics. In particular, Milgrom (1981) shows that the arrival of good (bad) news about a firm's prospects always leads to a rise (fall) in its share price. At the firm level, discontinuous up and down price movements may be a consequence of significant changes in the operating and financial structure of the firm, its competitive environment, changes in organization and other corporate plans.[7] Moreover, Kou (2002) argues that investors' sentiment in the form of under-reaction and overreaction, as documented in behavioral finance literature, leads to differential response to good and bad news. At the macroeconomics level, expansionary and contractionary periods are accompanied with unequal frequency of good and bad news arrivals (see Maheu & McCurdy (2004)).[8] The differential in intensity of news arrival may be in turn driven by broader economic cycles, recurrent technological change and innovation, or perhaps unexpected shifts in social, demographic, and political cycles.

The separation of good from bad news implies that the range of values for the random percentage change in price must be constrained. Because stocks represent limited liability, the percentage change in prices due to bad news must be bounded from below by minus one hundred percent. Similarly, the percentage change in prices due to arrival of good news must be positive. Because of these constraints, care must be taken when choosing a distribution for either up or down jump magnitudes. Under the PBJD, the jump magnitudes for good and bad news are drawn from the Pareto and Beta distributions, respectively. In addition to having the appropriate supports, these distributions lead to a tractable likelihood function and facilitate MLE.

Let $S(t)$ denote the price of stock at time t and assume that the price process can be represented by the following:

$$\frac{dS(t)}{S(t-)} = \mu dt + \sigma dZ(t) + \sum_{j=u,d} (V^j_{N^j(\lambda^j t)} - 1) dN^j(\lambda^j t) \tag{1}$$

where μ and σ are the drift and volatility terms, $Z(t)$ is a standard Wiener process, V^j is the jump magnitude, and $N^j(\lambda^j)$ are independent Poisson processes with intensity parameters λ^j ($j = u, d$ represent up- and down-jumps respectively). At this point it may be tempting to extend this specification to include stochastic volatility. However, given our objective to provide parameter estimates for DEJD we will not undertake this extension. Keppo, Meng, Shive & Sullivan (2003) have extend the DEJD to include stochastic volatility but these authors do not obtain MLE estimates for this specification. In general, it is difficult to obtain a closed form likelihood function for a model that nests jump structures like equation (1) and continuous time conditional heteroskedasticity. Moreover, as Huang & Wu (2004) observe, the variation in return volatility can be generated either by variations in diffusion variance, or variations in the arrival rates of jumps, or a combination of the two. These authors argue that the evidence favors variations in jump intensity, which is partially achieved by the formulation in equation (1).

It is assumed that the up-jump magnitudes (V^u) are distributed Pareto(η_u) with density function $f_{V^u}(x)$=($\frac{\eta_u}{x^{\eta_u+1}}$) where $V^u \geq 1$, E(V^u)=$\frac{\eta_u}{\eta_u-1}$ and var(V^u)=$\frac{\eta_u}{(\eta_u-2)(\eta_u-1)^2}$. Similarly, the down-jump magnitudes (V^d) are distributed Beta(η_d,1) with density function $f_{V^d}(x) = \eta_d x^{\eta_d-1}$ where $0 < V^d < 1$, E(V^d) $= \frac{\eta_d}{(\eta_d+1)}$ and var(V^d) $= \frac{\eta_d}{(\eta_d+2)(\eta_d+1)^2}$. All jumps are assumed to be independent, which implies a mixture of Pareto-Beta distributions for jump magnitudes. The specification in (1) is a Lévy process; it has stationary and independent increments and is continuous in probability.[9]

The Doléans-Dade formula (Protter 1991) provides an explicit solution for (1):

$$S(t) = S(0)\exp\{(\mu - \frac{1}{2}\sigma^2)t + \sigma Z(t)\} \prod_{j=u,d} V^j(N(\lambda^j t)) \tag{2}$$

$$\prod_{j=u,d} V^j(N(\lambda^j t)) = \begin{cases} 1 & \text{if } N(\lambda^j t) = 0 \\ \prod_{i=1}^{N(\lambda^j t)} V_i^j & \text{if } N(\lambda^j t) = 1, 2, 3, \ldots \end{cases}$$

Let $Y = ln(V)$, using equation (2), the s period rate of return, $r(s)$, is:

$$r(s) = (\mu - \frac{1}{2}\sigma^2)s + \sigma Z(s) + \sum_{i=1}^{N_s^u} Y_i^u + \sum_{i=1}^{N_s^d} Y_i^d \tag{3}$$

where $N_s^j, j = u, d$ denotes the number of good (bad) news over the time period s. In appendix 1 we derive both the conditional and the unconditional density functions for $r(s)$ and show that it is a probability weighted mixture of normal and exponential distributions.

The connection between the PBJD and DEJD can now be established as follows. Let $\lambda = \lambda_u + \lambda_d$, $p = \frac{\lambda_u}{\lambda_u+\lambda_d}$, and $q = 1 - p$, then the distribution of jump magnitudes is a probability weighted mixture of Pareto and Beta:

$$f_{\tilde{V}}(x) = p\frac{\eta_u}{x^{\eta_u+1}}\mathbf{I}_{\{x>1\}} + q\eta_d x^{\eta_d-1}\mathbf{I}_{\{0<x<1\}};\ \ \eta_u > 1, \eta_d > 0$$

where, setting $Y = ln(\tilde{V})$ and noting that the distribution of the logarithm of Pareto and Beta is exponential (see appendix 1), we obtain:

$$f_Y(y) = p\eta_u e^{-\eta_u y}\mathbf{I}_{\{y\geq 0\}} + q\eta_d e^{\eta_d y}\mathbf{I}_{\{y<0\}} \tag{4}$$

This is the distribution of the logarithm of the jump magnitudes under the DEJD, which has a jump intensity λ and Y has an IID mixture distribution of Exponential(η_u) and Exponential(η_d) with probabilities p and q, respectively. Note that from an inference perspective, both models have the same number of parameters to estimate: $\theta_{DEJD} = (\mu, \sigma, \lambda, p, \eta_u, \eta_d)$ and $\theta_{PBJD} = (\mu, \sigma, \lambda_u, \lambda_d, \eta_u, \eta_d)$. Having established the connection between these models, in the reminder of the paper we use "DEJD" to refer to both.

Merton's Log-normal Jump-Diffusion (LJD) model also has a single jump component with magnitude V distributed IID log-normal (α, β^2) and Poisson (λ) arrival rate. However, the LJD and DEJD specifications *are not* nested. Without the jump components, these models reduce to the standard GBM.

Table 1 presents the first four moments of returns for GBM, LJD, and the DEJD. Clearly, relative to GBM, both LJD and DEJD are capable of generating a higher peak, positive or negative skewness, and positive kurtosis and are therefore likely to better match the empirical returns distribution. However, a priori it is not clear whether DEJD performs better than LJD in this regard. An empirical examination is needed to address this issue. Focusing on the jump specification in (4), three special cases can be delineated:

(1). Suppose $\eta_u = \eta_d = \eta$ and $\lambda_u = \lambda_d = \lambda$ (i.e., $p = .5$), then the distribution of jumps will be symmetrical with higher peak and positive kurtosis relative to normal. Tsay (2002, page 245) discusses the properties of this special case and notes that for finite samples, it will be difficult to distinguish this distribution from the Student-t. Unlike the latter, however, the former is tractable analytically and can generate a higher probability concentration around its mean. As Tsay (2002), Huang & Huang (2003), and Carr & Wu (2004) show, this form of symmetry leads to simpler option pricing formula.[10] Absent actual parameter estimates, this assumption appears innocuous and has been commonly invoked.

(2). Suppose $\eta_u = \eta_d = \eta$ and $\lambda_u \neq \lambda_d$, then relative to GBM, the distribution of $r(s)$ will be skewed and have excess kurtosis and the relative size of λ_u and λ_d will lead to negative or positive skewness.

(3). Suppose $\eta_u \neq \eta_d$ and $\lambda_u = \lambda_d = \lambda$, again the resulting $r(s)$ will be skewed and show excess kurtosis. However, the relative size of η_u and η_d will determine whether the distribution is negatively or positively skewed.

Note that relative to LJD, the DEJD provides additional econometric flexibility in the following sense: Under LJD, a single compound Poisson process derives the news arrival and both up and down jumps have the same arrival intensity and jump distribution. Under the DEJD, however, the above special cases may be distinguished.

Both the LJD and the DEJD can explain the widely documented volatility smile in the option pricing literature. In particular, Kou (2002), Kou & Wang (2004), and others have demonstrated that the excess kurtosis and skewness of the DEJD under the physical measure generates similar features in the risk neutral distribution, leading to errors in option prices, particularly for deep in and out of money options, resulting in a convex implied volatility surface.[11]

The estimated DEJD parameters reported below indicate strong negative skewness in both the risk neutral and the physical returns distribution, which indicates that the probability of a large decrease in stock prices exceeds the probability of a large increase. Jackwerth & Rubinstein (1996) termed this phenomenon as “crashophobia”. The economic rationale for crashophobia is that put options are used as hedging instruments to protect against large downward movements in stock prices. This demand by investors due to portfolio insurance strategies has increased the price of protection (resulting in a “crash premium”) and therefore the left tail of the risk neutral distribution has more weight.

3 Maximum Likelihood Estimation

We rely on maximum likelihood estimation (MLE) method to obtain parameter estimates for DEJD because MLE has desirable statistical properties. Other methods for the estimation of jump-diffusion processes, including the generalized method of moments, the simulated moment estimation, and MCMC methods, among others are surveyed in Aït-Sahalia & Hansen (2004). The details on MLE estimation of jump-diffusion processes is discussed in Sorensen (1991), who proves that for large samples, MLE is the best method of estimation, because under mild regularity conditions, the estimated parameter are consistent, asymptotically normal and asymptotically efficient (also see Bates (2003*b*) and the reference section of Aït-Sahalia (2002)). However, MLE requires a complete specification of the transition density, which for nonlinear models may be difficult to obtain. Fortunately, the DEJD is a linear process with independent increments and an explicit transition density. Moreover, the selected distributions for the jump components have properties that make the MLE tractable. Lastly, Aït-Sahalia (forthcoming) has shown that MLE offers advantages in disentangling jumps from diffusion.

Let $D = \{S(0), S(1), S(2), \ldots, S(M)\}$ denote the the realizations of stock price at equally-spaced times $k = 0, 1, 2, \ldots, M$. The one period rate of return $r_i = \ln S(i) - \ln S(i-1)$ is IID. The unconditional density of $s = 1$ period returns, $f(r)$, is (see appendix 1 for derivation):

$$f(r) = e^{-(\lambda_u+\lambda_d)} f_{0,0}(r) + e^{-\lambda_u} \sum_{n=1}^{\infty} P(n, \lambda_d) f_{0,n}(r) + e^{-\lambda_d} \sum_{m=1}^{\infty} P(m, \lambda_u) f_{m,0}(r) + \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} P(n, \lambda_d) P(m, \lambda_u) f_{n,m}(r)$$

Where $P(n, \lambda) = \frac{e^{-\lambda}\lambda^n}{n!}$ and $f_{n,m}(r)$ ($n \geq 0$ and $m \geq 0$) is the conditional density for one period returns, conditional on the given numbers of up and down jumps (m, n). The log-likelihood given M equally spaced returns observations is:

$$L(D; \lambda_u, \lambda_d, \eta_u, \eta_d, \mu, \sigma^2) = \sum_{i=1}^{M} ln(f(r_i)) \tag{5}$$

The unconditional density, $f(r)$, is a mixture density (i.e., a Poisson weighted sum of four conditional densities). Kiefer (1978) and Honoré (1998) have shown that for mixture densities, care must be taken to ensure that the log-likelihood function remains bounded by properly restricting the admissible parameters. Otherwise a singularity problem arises, and the log-likelihood function becomes infinite. Hamilton (1994, page 689) and Kiefer (1978) have offered remedies to deal with this problem. As Hamilton (1994) shows singularities do not pose a major problem so long as the the selected numerical maximization procedure converges to a local maxima. Moreover, standard errors for the estimates can be constructed using the information matrix (see below).

The Newton-Raphson method has been a widely used numerical procedure for likelihood optimization. This method requires the first and second order derivatives of the log-likelihood function. Such derivatives are difficult to compute for the DEJD model. To avoid this difficulty we use Powell's method. The latter method uses successive line optimization in the conjugate directions and does not necessitate the use of the derivatives. Hamilton (1994, page 139) provides a complete description of Powell's optimization procedure. The optimization programs we use are taken from Press, Teukolsky, Vetterling & Flannery (1992).

The likelihood function in (5) involves double infinite summations and double improper integrals. First, piecewise Gaussian quadratures are employed to compute the integrals (Press et al. 1992). We find that for plausible parameter values, lower bound truncation of the integrals at (-2.0) provides six digit accuracy for most cases. Next, the infinite sums are calculated using the usual termination criterion; if $S_n = \sum_{i=1}^{n} X_i$, then we stop the summation if $2|X_{n+1}| \leq FTOL \times (|S_n| + |S_{n+1}|)$. We choose $FTOL = 10^{-10}$, which guarantees at least eight digits accuracy. Standard error for the estimates is obtained by the *outer-product* method, which is based on the first derivative of the likelihood function (see Hamilton (1994), page 143). We performed extensive Monte Carlo simulation and found that our programs and the estimation procedures are accurate and reliable. The Monte Carlo results will not be reported here to save space but are available from the authors.

4 Model Selection

The DEJD is compared with the LJD, the GBM, and six popular versions of ARCH specification.[12] For model selection, we adopt the widely-used Bayesian Information Criterion (BIC) proposed by Schwarz (1978). Unlike significance tests, BIC allows comparison of more than two models at the same time and does not require that the alternatives to be nested. BIC is a "conservative" criterion (relative to AIC) in the sense that it heavily penalizes over parameterization.

Suppose the kth model M_k, has parameter vector θ_k, where θ_k consists of n_k independent parameters to be estimated. Denote $\hat{\theta}_k$ as the MLE of θ_k. Then, BIC for Model M_k is defined as:

$$BIC_k = -2 \log f(D|\hat{\theta}_k, M_k) + n_k \log(m),$$

where m is the number of observations in data set D and $f(D|\hat{\theta}_k, M_k)$ is the maximized likelihood function. Clearly the best "fit" model is one with the smallest BIC.

We provide comparisons of the DEJD ($\theta_{DEJD} = (\mu, \sigma, \lambda, p, \eta_u, \eta_d)$) relative to GBM ($\theta_{GMB} = (\mu, \sigma)$), the LJD ($\theta_{JD} = (\lambda, \alpha, \beta, \mu, \sigma)$) and the following popular ARCH specifications. Suppose return over the i-th period, $r_i = \mu + \varepsilon_i$, where $\varepsilon_i = \sqrt{h_i} e_i$ and $\{e_i\}$ is a sequence of draws from an IID standard normal. Six ARCH alternatives are obtained by different parameterization of h_i:

(1). ARCH(1): $h_i = \omega + \alpha_1 \varepsilon_{i-1}^2$ and $\theta_{ARCH(1)} = (\mu, \omega, \alpha_1)$.

(2). ARCH(2): $h_i = \omega + \alpha_1 \varepsilon_{i-1}^2 + \alpha_2 \varepsilon_{i-2}^2$ and $\theta_{ARCH(2)} = (\mu, \omega, \alpha_1, \alpha_2)$.

(3). GARCH(1,1): $h_i = \omega + \alpha_1 \varepsilon_{i-1}^2 + \gamma_1 h_{i-1}$ and $\theta_{GARCH(1,1)} = (\mu, \omega, \alpha_1, \gamma_1)$.

For EGARCH models, set $z_i = \varepsilon_i / h_i$, and $g(z_i) = \rho z_i + [|z_i| - E|z_i|]$. Then,

(4). EGARCH(1): $\ln(h_i) = w + \alpha_1 g(z_{i-1})$ and $\theta_{EGARCH(1)} = (\mu, w, \alpha_1, \rho)$.

(5). EGARCH(2): $\ln(h_i) = w + \alpha_1 g(z_{i-1}) + \alpha_2 g(z_{i-2})$ and $\theta_{EGARCH(2)} = (\mu, w, \alpha_1, \alpha_2, \rho)$.

(6). EGARCH(1,1): $\ln(h_i) = w + \alpha_1 g(z_{i-1}) + \gamma_1 \ln(h_{i-1})$ and $\theta_{EGARCH(1,1)} = (\mu, w, \alpha_1, \gamma_1, \rho)$.

5 Data and Results

The estimation of DEJD, as well as the calculation of the standard errors, is computationally expensive and time consuming because the likelihood function involves double infinite summations and double improper integrals. On a high-performance computer (Itanium II, 1.3 GHz CPU with 1GB RAM using HP-Fortran), 4-7 hours of computation time is needed to obtain the first set of parameter estimates for a series consisting of 1256 observations. To ensure the likelihood is fully optimized, at least two rounds of estimation are conducted for each series. While "good" initial parameter values may be helpful, and in some cases such values were obtained by cumulant matching methods, they are not necessary.

Another challenge arising during the estimation process is the possibility that the likelihood function may explode rather than converge. To avoid the singularity problem described above, we choose a range of initial values to ensure the parameter space is large enough to cover the true parameter values. We also ensure that the likelihood function obtained by Powell's method converges. Hence the conditions described in Hamilton (1994) and Kiefer (1978) are met and the consistency and asymptotic normality of the obtained maximum likelihood estimates are guaranteed.

To manage the required computational time, we adopt the following estimation strategy. As a first step, we select daily returns for 100 firms from the Center for Research in Security Prices (CRSP) data. These returns span the period 31/10/1996 through 31/12/1998 (547 Observations) and are adjusted for dividend and splits. We estimate GBM, LJD, and DEJD for each series and use the BIC criterion to identify firms for which the DEJD specification fits the data best. In the second step, we use longer and more recent data for the "best fit" firms, as well as daily and monthly data for the S&P-500 and the NASDAQ composite to assess the performance of DEJD.

The 100 firms are selected for the first step of this analysis trade on the main exchanges in the United States (48 NASDAQ, 48 NYSE, and 4 AMEX and OTC). The criteria for selecting these firms are as follows. Roughly half of the sampled firms are well recognized names, such as General Electric (GE) and Microsoft (MSFT). These firms were chosen because they permit comparisons to other studies. Moreover, a large number of analysts and institutional investors follow and trade on the news affecting these highly liquid firms, which is important given the event driven nature of the jump-diffusion models. The other half of the sampled firms were selected for their high kurtosis.

Table 2 contains the log-likelihood values for GBM, LJD, and DEJD specifications, as well as the estimated parameters for DEJD. Two clear results emerge from the data in the table. First, based on BIC criterion, both the LJD and DEJD fit the data better than GBM for every firm. This is not surprising as it confirms the findings reported in previous studies that favor models with jumps. Second, and perhaps somewhat surprising, the DEJD performs better than LJD for only 11 firms (6 Nasdaq and 5 NYSE). Of course, the weak support for DEJD specification may be a consequence of the time period considered, the firms sampled, the up and down distributions selected, or the choice of BIC, which is a relatively conservative criterion. Nonetheless, the results fall short of our expectations given the flexibility associated with DEJD.

Table 2 (last 5 lines) also contains summary statistics on the parameter estimates for the

DEJD. On average, it appears that the intensity of good news arrival is about once every two days ($\lambda_u = 0.49$), resulting in an average up jump of 1.89% ($\frac{\eta_u}{\eta_u - 1}$). For the down jumps these figures are once every three days and 2.00% ($\frac{\eta_d}{\eta_d + 1}$), respectively. The average jump amplitudes seem relatively small, while the intensity of news arrival appears large.[13] It is important to note that the combination of high jump intensity and small jump magnitudes generates high peak and the leptokurtic feature under DEJD.

Overall, the parameter estimates reported in Table 2 have reasonable values and are informative. In particular, the table shows that the symmetric version of DEJD (i.e., $\eta_u = \eta_d = \eta$ and $\lambda_u = \lambda_d = \lambda$), rarely occurs. As noted earlier, several important applications of the DEJD, including Huang & Huang (2003), Broadie & Yamamoto (2003), and Carr & Wu (2004), invoke this assumption. The data in Table 2 provides no empirical justification for this assumption.

In the second step of this analysis, we assess the performance of the DEJD in relation to the GBM, LJD, and the ARCH alternatives. For this purpose, we use daily returns data spanning the period 1/1999 through 12/2003 ($N = 1256$) for the 11 firms that emerge from the first step of our analysis. We conduct similar comparisons for daily and monthly (value weighted) returns for the S&P-500 and the NASDAQ composite index. Four S&P-500 return series are utilized: SPD1 (SPD2) is the daily raw (dividend adjusted) returns for the period 7/1962 through 12/2003 ($N = 10446$). The monthly series, SPM1 and SPM2, are similarly defined but span the period 1/1926 through 12/2003 ($N = 936$). The NASDAQ series span the period 1/1973 through 12/2003: NASD ($N = 7828$) and NASM ($N = 372$) represent the daily and monthly returns respectively.[14]

Table 3 presents the sample statistics for these returns data. The large range of returns, particularly for the indexes, reflect significant booms and crashes that occur during the period considered. All returns appear highly skewed and exhibit large tail probabilities, and not surprisingly, the jump specifications provide reasonably good fit for these data. Finally, note that the skewness and kurtosis of individual stocks is similar to that of the indexes.

Table 4 reports the BIC values for nine alternative specifications. As noted above, the model with the smallest BIC provides the best fit to the data. The results in the table can be summarized as follows: Comparing DEJD to LJD, we find that DEJD performs better for 7 individual stocks and all indexes except NASM. The jump diffusion specifications perform better than the ARCH alternatives for the majority of individual stocks, especially for T, HYBD, LCBM, MFRI, TBL, and XICO, where the BIC associated with DEJD is considerably larger than the ARCH alternatives in absolute value. For indexes the ARCH specifications dominate. More precisely, the GARCH(1,1) dominate for monthly indexes and EGARCH(1,1) for daily indexes. These results favoring ARCH are consistent with findings reported in other studies (see Gourieroux (1997)). The GBM specifications fails to beat the alternatives for every returns series.

The MLE estimates for GBM, LJD, and DEJD are presented in Table 5. Parameter estimates that are significant at 95% or higher confidence level appear in bold face. Under the DEJD, all parameter estimates for the daily S&P-500 and daily NASDAQ returns are statistically significant and their magnitudes confirm the widely held view that indexes tend to jump down more than they jump up. The results for individual stocks appear somewhat mixed; while the estimated mean down-jump (η_d) is always significant, the associated jump intensity (λ_d) is rarely significant. Under the LJD, the estimated jump intensity (λ) is always significant while the mean jump size (α) is mostly insignificant. As expected, the addition of a jump component significantly changes the estimated drift and volatility parameters associated with the continuous part of the process.

The parameter estimates in Table 5 in conjunction with the formula in Table 1 can be used to calculate the moments of the returns distribution. Comparison of the calculated moments with the sample moments, particularly skewness and kurtosis (not tabulated), shows that the DEJD matches the empirical moments quite well. Utilizing the parameter estimates in Tables 5, in Figure 1 we plot the fitted densities for GBM, LJD, and DEJD against the sample histograms for the S&P-500 and the NASDAQ daily and monthly series. Clearly the DEJD offers significant improvement in matching the high peak, skewness, and large kurtosis of returns.

Finally, Table 6 presents the parameter estimates for GBM, LJD, and DEJD for monthly S&P-500 returns for the period 1926-2003 and several sub-periods. The table shows that both the jump intensity and the distribution of jump magnitudes associated with LJD and DEJD change considerably over time, though the shift in parameters appear to be consistent with the economic events and trends in each sub-period. The data in Table 6 indicate significant parameter instability, particularly time-varying jump intensity. Andersen, Benzoni & Lund (2002) extend the JD models to allow for time-varying jump intensity. Applying this generalization to the DEJD represents an interesting extension, which the authors are pursuing at this time.

6 Conclusions

The paper presented the first comprehensive empirical assessment of the DEJD. We provided maximum likelihood estimates for a large sample of CRSP firms, the S&P-500, and the NASDAQ composite. Using the BIC criterion, we assessed the performance of DEJD relative to six popular versions of ARCH, the LJD, and GBM. We found that while DEJD is superior to GBM, its performance relative to LJD and the ARCH alternatives is considerably weaker. These finding are significant because while the LJD is simpler from an econometric perspective (transition density has a convenient expression), the DEJD is better suited for pricing derivatives, particularly path-dependent contingent claims (See Kou (2002), Kou & Wang (2004), and Sepp (2004)). Accordingly, it is interesting to determine which model is more compatible with index and individual equity data. Our results shows that for indexes, the DEJD performs better than LJD, though both are dominated by ARCH alternatives. For individual stocks, the DEJD performs poorly overall, except for firms marked with"****" in Table 5. Therefore, we conclude that care should be taken when applying the DEJD option pricing formula to price individual stock options.

The weak support for DEJD specification may be a consequence of the distributional choice for the jump magnitudes. The choice of jump magnitude distribution may be particularly important. To see this contention, note that the estimates for η_u in Table 2 ranges from 14 to 150, with the majority exceeding 40. Given these parameter values, the density function for the up jumps, $f_{V^u}(x)$=($\frac{\eta_u}{x^{\eta_u+1}}$), appears to peak at 1 (i.e., a price jump of 0%), and drops sharply as x increases. For example, if $\eta_u = 80$, then over 95% of the up jumps will be less than 3% in magnitude. For individual stocks, price jumps of less than 3% probably should perhaps be regarded as normal part of the GBM. Therefore, for most price increases, the DEJD specification will have a difficult time separating normal price movements from jumps. Perhaps a better choice of the distribution should have a single mode above 3%. Given this observation, future research should consider other distributions such as Gamma. A better distributional choice will likely enhance the empirical support for this generalization, though the likelihood function is bound to become more complicated.

There are a number of other interesting directions for future extensions of this work. As a starting point, other estimation techniques, such as the generalized method of moments and its variants, may be utilized. As Eraker et al. (2003) and others have shown, stochastic volatility is an important component of the return process and should be integrated into the DEJD specification. This extension has already been proposed by Keppo et al. (2003) but it results in a very complicated likelihood function. With the moment based methods it may be simpler to determine whether stochastic volatility remains important when the jump component of return process has a more complex structure as in DEJD. Time-varying jump intensities offer another direction to extend the DEJD specification. In related research the authors are currently investigating some of these issues.

On a theoretical front, optimal inter-temporal portfolio choice models (e.g., Wu (2003)) can be revisited taking the DEJD process as exogenous. In similar vein, information arrival plays a significant role in driving the dynamics of other economic variables, including foreign exchange, inflation, short term interest rates, and commodity prices. The DEJD can be easily adopted to these areas as well. Moreover it may be interesting to assess the significance of the proposed distinction between good and bad news in these settings.

Appendix 1: The Derivation of the Density of Returns

Let $N_s^u = m$ and $N_s^d = n$ be the number of up- and down-jumps during the time interval with length s. The conditional densities of s period returns can be derived under four combination of m and n: $m = 0$ and $n = 0$ (no jumps occur); $m = 0$ and $n \geq 1$ (only down-jumps occur); $m \geq 1$ and $n = 0$ (only up-jumps occur); and $m \geq 1$ and $n \geq 1$ (both types of jumps occur). All four conditional densities can be derived using convolution techniques and distributional properties. Before deriving the conditional density, we note some useful facts about Pareto, Beta and exponential distributions (Patel, Kapadia & Owen 1976):

F1. If $V^u \sim$ Pareto (η_u), then $Y^u = \ln(V^u) \sim \exp(\eta_u) = \Gamma(1, \eta_u)$.
F2. If $V^d \sim$ Beta $(\eta_d, 1)$, then $-Y^d = -\ln(V^d) \sim \exp(\eta_d) = \Gamma(1, \eta_d)$.
F3. If $X = Y_1 + Y_2 + \cdots + Y_n$, where $Y_i \sim \exp(\theta)$ and they are independent, then $X \sim \Gamma(n, \theta)$.

Let $U = \sum_{i=1}^{N_s^u} Y_i^u > 0$, $D = \sum_{i=1}^{N_s^d} Y_i^d < 0$ and $T = U + D$. Then s period return can be written as $r(s) = (\mu - 0.5\sigma^2)s + Z(s) + U + D$. For $N_s^u = m \geq 1$ the conditional distribution of U (by F1 and F3) is $U|m \sim \Gamma(m, \eta_u)$ with the density:

$$f_{U|m}(U) = \frac{\eta_u^m}{(m-1)!} U^{m-1} e^{-\eta_u U}$$

Similarly, for $N_s^d = n \geq 1$ the conditional distribution of D is $-D|n \sim \Gamma(n, \eta_d)$ with the density:

$$f_{D|n}(D) = \frac{\eta_d^n}{(n-1)!} (-D)^{n-1} e^{\eta_d D}$$

Applying the above two results, the conditional density of $T = U + D$, given $m \geq 1$ and $n \geq 1$, is:

$$\begin{aligned} f_{T|m,n}(t) &= \int_{-\infty}^{\infty} f_D(x) f_U(t-x) dx \\ &= \frac{\eta_u^m \eta_d^n e^{-\eta_u t}}{(m-1)!(n-1)!} \int_{-\infty}^{0 \wedge t} (-x)^{n-1} (t-x)^{m-1} e^{(\eta_u + \eta_d)x} dx \end{aligned} \tag{A1}$$

Now, we are ready to determine all four conditional densities. For the case $m = 0$ and $n = 0$, the conditional density is that of $N((\mu - 0.5\sigma^2)s, \sigma^2 s)$:

$$f_{r(s)|0,0}(r) = \frac{1}{\sqrt{2\pi s}\sigma} e^{-\frac{1}{2\sigma^2 s}(r-\mu s+0.5\sigma^2 s)^2} \quad \text{(A2)}$$

When $m = 0$ and $n \geq 1$, the conditional distribution is the independent sum of $-\Gamma(n, \eta_d)$ and N($(\mu - \frac{1}{2}\sigma^2)s$, $\sigma^2 s$):

$$f_{r(s)|0,n}(r) = \frac{\eta_d^n}{(n-1)!\sqrt{2\pi s}\sigma} \int_{-\infty}^{0} (-x)^{n-1} e^{\eta_d x - \frac{1}{2\sigma^2 s}(r-x-\mu s+0.5\sigma^2)^2 s} dx \quad \text{(A3)}$$

Similarly, for $m \geq 1$ and $n = 0$, the conditional distribution is the independent sum of $\Gamma(m, \eta_u)$ and N($(\mu - \frac{1}{2}\sigma^2)s$, $\sigma^2 s$):

$$f_{r(s)|m,0}(r) = \frac{\eta_u^m}{(m-1)!\sqrt{2\pi s}\sigma} \int_{0}^{\infty} (x)^{m-1} e^{-\eta_u x - \frac{1}{2\sigma^2 s}(r-x-\mu s+0.5\sigma^2 s)^2} dx \quad \text{(A4)}$$

Finally, for $m \geq 1$ and $n \geq 1$, the conditional distribution is the independent sum of the distribution for T and N($(\mu - \frac{1}{2}\sigma^2)s$, $\sigma^2 s$). Then the conditional density of $r(s)$ is:

$$\begin{aligned} f_{r(s)|m,n}(r) = {} & \frac{\eta_u^m \eta_d^n}{(m-1)!(n-1)!\sqrt{2\pi s}\sigma} \\ & \int_{-\infty}^{\infty} \left(\int_{-\infty}^{0\wedge t} (-x)^{n-1}(t-x)^{m-1} e^{(\eta_u+\eta_d)x} dx \right) \times e^{-\eta_u t} e^{-\frac{1}{2\sigma^2 s}(r-t-\mu s+0.5\sigma^2 s)^2} dt \quad \text{(A5)} \end{aligned}$$

Next we derive the unconditional density of $s = 1$ period returns. Letting $P(n, \lambda) = \frac{e^{-\lambda}\lambda^n}{n!}$, the unconditional density for one period returns, $f(r)$, can be written as the Poisson weighted sum of the four conditional densities (A2-A5):

$$\begin{aligned} f(r) &= \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} P(n, \lambda_d) P(m, \lambda_u) f_{n,m}(r) \\ &= e^{-(\lambda_u+\lambda_d)} f_{0,0}(r) + e^{-\lambda_u} \sum_{n=1}^{\infty} P(n, \lambda_d) f_{0,n}(r) \\ &\quad + e^{-\lambda_d} \sum_{m=1}^{\infty} P(m, \lambda_u) f_{m,0}(r) + \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} P(n, \lambda_d) P(m, \lambda_u) f_{n,m}(r) \quad \text{(A6)} \end{aligned}$$

and as equation (A6) shows, the unconditional distribution of returns is a *mixture density*.

Table 1: **Moments of returns.** The first four moments of s period returns ($r(s)$) under GBM and two Jump-Diffusion specifications. The second and third moments are defined as Skewness $= E(X - EX)^3/[Var(X)]^{3/2}$ and Kurtosis $= E(X - EX)^4/[Var(X)]^2 - 3$.

	GBM	JD	DEJD
E$r(s)$	$(\mu - \frac{1}{2}\sigma^2)s$	$(\mu - \frac{1}{2}\sigma^2 + \lambda\alpha)s$	$(\mu - \frac{1}{2}\sigma^2 + \frac{\lambda_u}{\eta_u} - \frac{\lambda_d}{\eta_d})s$
Var$[r(s)]$	$\sigma^2 s$	$(\sigma^2 + \lambda(\beta^2 + \alpha^2))s$	$(\sigma^2 + 2\frac{\lambda_u}{\eta_u^2} + 2\frac{\lambda_d}{\eta_d^2})s$
Skewness	0	$\frac{\lambda\alpha^3}{(\sigma^2+\lambda(\beta^2+\alpha^2))^{1.5}s^{0.5}}$	$\frac{6(\frac{\lambda_u}{\eta_u^3}-\frac{\lambda_d}{\eta_d^3})}{(\sigma^2+2\frac{\lambda_u}{\eta_u^2}+2\frac{\lambda_d}{r_d^2})^{1.5}\sqrt{s}}$
Kurtosis	0	$\frac{\lambda(\alpha^4+3\beta^4)}{(\sigma^2+\lambda(\beta^2+\alpha^2))^2 s}$	$\frac{24(\frac{\lambda_u}{\eta_u^4}+\frac{\lambda_d}{\eta_d^4})}{(\sigma^2+2\frac{\lambda_u}{\eta_u^2}+2\frac{\lambda_d}{\eta_d^2})^2 s}$

Table 2: **Log-likelihood for alternative models.** Dividend and split adjusted returns data for 100 firms are used to estimate three alternative specifications: the Geometric Brownian Motion (GBM), the Log-normal Jump Diffusion (LJD), and the Double Exponential Jump Diffusion (DEJD). The daily data spans the period 10/96 through 12/98 (N=547). The first three columns report the log-likelihood for each specification. Based on BIC criterion, models with jumps fit the data better than GBM for all firms. The cases where DEJD performs better than LJD (11 firms) appear in bold. The results for daily S&P500 and summary statistics for the estimated parameters are reported at the bottom of this table.

Stock:Ticker	Log-Likelihood			Parameter Estimates for DEJD					
	GBM	LJD	DEJD	λ_u	λ_d	η_u	η_d	μ	σ
APPLE COMP:AAPL	999.76	1079.65	1082.19	0.31	0.23	30.38	38.52	-0.002	0.020
ABIOMED INC:ABMD	918.36	944.22	943.96	0.55	0.67	34.70	52.66	-0.003	0.026
KONINK AHOLD:AHO	1339.58	1360.30	1360.17	0.54	0.32	80.98	103.66	-0.002	0.015
AMGEN:AMGN	1300.43	1332.07	1331.69	0.91	0.24	91.15	66.68	-0.005	0.013
AMR CORP:AMR	1285.69	1301.73	1300.88	0.23	0.17	65.69	53.11	0.001	0.018
ACT NET:ANET	867.87	909.61	910.53	0.71	0.25	32.11	31.75	-0.015	0.025
AOL-TIME WRNR:AOL	998.07	1026.66	1026.84	0.26	0.03	31.47	21.03	0.000	0.030
APPLIED MATL:AMAT	1007.35	1018.62	1017.76	0.09	0.41	27.98	75.34	0.005	0.033
ATT CP:T	1361.81	1390.77	1394.14	0.75	0.04	108.16	29.60	-0.004	0.014
BOEING CO:BA	1279.95	1351.47	1353.32	0.16	0.06	47.56	27.93	-0.001	0.016
BANK OF AMERICA:BAC	1293.71	1315.74	1315.69	0.16	0.29	57.12	61.65	0.003	0.017
BIOGEN IDEC:BIIB	1113.20	1130.73	1131.07	0.25	0.44	43.68	60.22	0.000	0.022
BIOSOURCE INTL:BIOI	988.18	1035.97	1036.71	0.38	0.51	33.39	43.93	-0.005	0.019
BIO LOGIC:BLSC	713.37	772.81	775.48	0.60	0.93	21.79	37.36	-0.001	0.020
BMC SOFTWARE:BMC	1057.16	1078.03	1079.93	0.59	0.06	54.43	24.31	-0.006	0.025
BIOMET INC:BMET	1183.47	1209.94	1208.72	0.37	0.23	50.39	62.99	0.001	0.019
BIOMERICA:BMRA	764.20	801.41	802.07	0.66	0.92	25.98	42.37	-0.004	0.025
BIO-REF LABS:BRLI	730.49	762.58	764.10	0.83	1.06	27.76	39.73	-0.002	0.023
BIO VASCULAR:BVAS	978.02	1016.43	1016.95	0.34	0.81	29.90	65.59	0.001	0.023
BRE-X MINE:BXM	988.25	1003.36	1004.06	0.81	0.18	53.94	34.48	-0.006	0.027
O'CHARLEY'S:CHUX	1089.95	1157.09	1159.54	0.33	0.52	37.56	62.48	0.001	0.017
CISCO SYS:CSCO	1164.95	1180.59	1180.81	0.81	0.20	87.15	45.39	-0.002	0.020
CITIGROUP:C	1178.48	1213.89	1214.01	0.20	0.41	40.91	63.42	0.003	0.018
CMGI INC:CMGI	780.05	824.11	824.61	0.64	0.49	26.78	33.62	-0.002	0.027
COMPAQ CP:CPQ	1109.77	1117.80	1117.38	0.96	0.12	74.11	44.09	-0.007	0.024
COMPUTER ASSO:CA	1054.06	1117.35	1117.87	0.04	0.04	28.22	14.11	0.002	0.028
COMPUTER SCI:CSC	1236.12	1260.63	1262.40	0.43	0.07	66.66	33.10	-0.003	0.019
CYPRESS BIO:CYPB	830.28	906.73	912.69	0.48	0.89	24.56	49.08	0.001	0.019
DELTA AL:DAL	1317.34	1320.8	1322.66	0.30	0.11	83.75	71.82	-0.001	0.019
DELL INC:DELL	1074.43	1078.98	1081.21	0.52	0.03	91.57	24.62	0.002	0.030
DNLDSON LUF JEN:DLJ	1122.23	1168.44	1168.19	0.35	0.27	41.47	48.67	0.000	0.019
EMC CP:EMC	1093.70	1117.73	1119.2	0.92	0.05	58.96	30.76	-0.010	0.021
ENZO BIO:ENZ	1064.20	1140.34	1142.62	0.52	0.17	40.11	36.34	0.008	0.016
FEDEX CP:FDX	1287.68	1304.01	1302.94	0.39	0.24	66.93	75.72	-0.001	0.017
GENERAL ELEC:GE	1433.10	1449.48	1448.84	0.05	0.08	45.96	66.17	0.002	0.015
GRIFFON CP:GFF	1243.75	1290.48	1292.22	0.07	0.17	28.24	50.02	0.002	0.018

Table 2: Cont. **Log-likelihood for alternative models.** Dividend and split adjusted returns data for 100 firms, selected from CRSP, are used to estimate three alternative specifications: the Geometric Brownian Motion (GBM), the Log-normal Jump Diffusion (LJD), and the Double Exponential Jump Diffusion (DEJD). The daily data spans the period 10/96 through 12/98 (N=547). The first three columns report the log-likelihood for each specification. Based on BIC criterion, models with jumps fit the data better than GBM for all firms. The cases where DEJD performs better than LJD (11 firms) appear in bold. The results for daily S&P500 and summary statistics for the estimated parameters are reported at the bottom of this table.

Stock:Ticker	Log-Likelihood			Parameter Estimates for DEJD					
	GBM	LJD	DEJD	λ_u	λ_d	η_u	η_d	μ	σ
GOODRICH CP:GR	1342.30	1385.10	1384.89	0.07	0.11	37.42	50.05	0.001	0.016
GATEWAY INC:GTW	1022.66	1031.78	1029.50	0.69	0.24	56.19	52.13	-0.005	0.028
HEWLETT-PACKARD:HPQ	1260.45	1288.32	1291.36	1.33	0.18	100.35	49.26	0.008	0.013
H & R BLOCK:HRB	1417.94	1430.56	1431.21	0.46	0.05	107.78	50.04	0.002	0.015
HYCOR BIO:HYBD	828.58	889.94	1029.03	0.56	0.65	26.02	28.75	0.000	0.003
INTL BUS MACHINE:IBM	1345.97	1365.15	1365.51	0.22	0.05	67.02	43.57	0.000	0.017
INTEL CORP:INTC	1237.18	1249.91	1253.18	0.13	0.02	49.20	30.61	0.000	0.022
JPMORGAN CHASE:JPM	1304.60	1321.59	1321.35	0.21	0.17	61.29	61.70	0.000	0.017
KOGER EQUITY:KE	1445.12	1471.05	1469.21	0.34	0.13	86.20	68.32	-0.001	0.013
KOMAG INC:KMAG	854.20	931.85	933.85	0.36	0.30	24.67	29.21	-0.005	0.024
LIFECORE BIO:LCMB	933.43	1015.31	1021.60	0.41	0.87	26.33	53.91	0.001	0.013
LEHMAN BROS:LEH	1058.13	1126.77	1126.86	0.26	0.32	33.15	39.40	0.002	0.018
LEXMARK:LXK	1122.24	1178.06	1180.35	0.35	0.19	44.3	41.57	-0.001	0.018
LUCENT:LU	1218.75	1226.31	1226.86	0.38	0.03	78.19	35.09	0.000	0.022
MARTEK BIO:MATK	790.47	831.48	833.89	0.61	0.16	29.04	21.00	-0.013	0.032
MCI COMM CORP:MCIC	1252.10	1263.71	1266.47	0.95	0.13	96.53	51.61	-0.005	0.017
MERCK CO:MRK	1395.11	1413.93	1414.18	0.15	0.02	75.24	32.89	0.001	0.016
MERRILL LYNCH:MER	1149.34	1171.86	1171.32	0.38	0.20	54.03	45.76	-0.001	0.021
MFRI INC:MFRI	1066.87	1109.04	1124.80	0.82	0.94	48.65	56.91	-0.001	0.004
MICRON TECH:MU	968.48	977.17	975.96	0.88	0.24	52.72	47.42	-0.009	0.030
MINDSPRING:MSPG	854.66	873.59	872.36	0.83	0.58	38.72	41.63	0.000	0.029
MORGAN STNLY:MNDX	1149.08	1187.68	1188.20	0.45	0.19	52.08	38.40	-0.001	0.018
MOTOROLA:MOT	1233.49	1245.61	1246.52	0.34	0.07	70.11	38.90	-0.002	0.020
MICROSOFT :MSFT	1302.03	1307.71	1307.81	0.31	0.02	79.08	41.72	-0.001	0.020
MONSANTO:MON	1188.92	1251.98	1255.20	0.40	0.04	75.41	17.76	-0.002	0.019
NEXTEL COMM:NXTL	1021.93	1049.13	1051.06	0.75	0.07	47.78	27.89	-0.011	0.024
NEWBRIDGE NET:NN	1017.71	1053.04	1055.74	0.62	0.12	49.41	25.54	0.007	0.024
NOODLE KID:NKID	805.17	832.15	831.81	0.61	0.50	29.18	36.50	-0.004	0.031
NAPRO BIO:NPRO	650.72	749.12	751.53	0.29	0.13	14.33	14.98	-0.011	0.037
NORSTAN INC:NRRD	1171.82	1185.72	1188.00	0.82	0.36	75.23	57.39	-0.004	0.017
NORTEL NET:NT	1196.59	1251.39	1252.86	0.26	0.08	50.20	27.85	-0.001	0.018
NORTHWEST AL:NWAC	1119.87	1142.99	1141.25	0.38	0.65	42.80	87.75	-0.001	0.021
ODETICS INC:ODETA	837.64	882.80	885.50	0.49	0.10	33.22	17.46	-0.007	0.033
OPTICAL DATA:ODSI	848.01	913.35	915.59	0.41	0.62	24.03	45.25	-0.005	0.024
ONEOK INC:OKE	1428.45	1439.24	1439.57	0.44	0.21	103.61	100.49	-0.001	0.014
ONHEALTH NET:ONHN	838.52	866.35	865.48	0.57	0.62	29.86	39.00	-0.001	0.027
ONE PRICE CLTH:ONPR	805.76	847.57	848.46	0.57	0.73	27.38	40.45	0.000	0.025
PATRICK INDS:PATK	1284.11	1322.66	1323.53	0.25	0.18	52.34	62.02	-0.001	0.016
KON PHILIPS:PHG	1181.55	1199.20	1201.77	1.08	0.48	81.20	63.74	-0.004	0.015

Table 2: Cont. **Log-likelihood for alternative models.** Dividend and split adjusted returns data for 100 firms, selected from CRSP, are used to estimate three alternative specifications: the Geometric Brownian Motion (GBM), the Log-normal Jump Diffusion (LJD), and the Double Exponential Jump Diffusion (DEJD). The daily data spans the period 10/96 through 12/98 (N=547). The first three columns report the log-likelihood for each specification. Based on BIC criterion, models with jumps fit the data better than GBM for all firms. The cases where DEJD performs better than LJD (11 firms) appear in bold. The results for daily S&P500 and summary statistics for the estimated parameters are reported at the bottom of this table.

Stock:Ticker	Log-Likelihood			Parameter Estimates for DEJD					
	GBM	LJD	DEJD	λ_u	λ_d	η_u	η_d	μ	σ
PFIZER INC:PFE	1310.64	1316.06	1316.38	0.28	0.03	81.42	44.92	0.000	0.019
PAULSON CAP:PLCC	898.15	935.57	935.99	0.92	0.45	41.15	38.85	-0.009	0.022
PEOPLESOFT:PSFT	973.69	1023.77	1026.75	0.62	0.14	46.41	24.07	0.007	0.023
SUN MICRO:SUNW	1122.00	1125.40	1125.32	0.38	0.19	77.51	55.07	0.001	0.027
TAYLOR DEV:TAYD	1055.05	1139.6	1071.48	1.04	0.73	66.26	52.42	-0.003	0.017
TIMBERLAND:TBL	1159.87	1203.75	1208.18	0.12	0.67	28.95	81.84	0.005	0.019
TAUBMAN CNTR:TCO	1586.68	1604.25	1604.75	0.70	0.21	150.68	107.81	-0.002	0.009
TIDEWATER:TDW	1133.85	1148.13	1147.63	0.75	0.66	64.17	80.16	-0.004	0.019
TEXAS INST:TXN	1103.46	1111.20	1112.65	1.31	0.04	78.05	29.72	-0.012	0.023
TIFFANY:TIF	1224.89	1266.84	1271.46	0.33	0.20	51.68	58.54	-0.002	0.017
TYCO INTL:TYC	1359.59	1388.15	1388.04	0.16	0.05	62.00	38.55	0.001	0.016
UAL CP:UAL	1245.90	1254.42	1252.21	0.49	0.08	79.47	81.90	-0.004	0.021
UNIV ELECT:UEIC	1050.38	1102.11	1102.45	0.20	0.06	28.73	24.04	-0.003	0.024
UNIV HEALTH:UHS	1395.65	1413.66	1413.83	0.62	0.29	101.09	81.82	-0.001	0.012
UNITEDHLT:UNH	1156.51	1242.67	1248.61	0.37	0.05	56.20	18.61	-0.003	0.018
UNIT CP:UNT	1520.85	1546.91	1541.89	0.04	0.21	49.07	109.21	0.002	0.013
UNIVRSL ST:USAP	1144.69	1206.50	1208.37	0.15	0.25	29.87	48.22	0.001	0.018
UNIVERSAL CP:UVV	1416.64	1431.08	1430.82	0.11	0.17	58.41	84.05	0.001	0.015
US WIRELESS:UWR	1440.66	1474.10	1473.62	0.16	0.74	56.36	162.57	0.003	0.012
WAL-MART:WMT	1350.98	1362.86	1362.39	0.15	0.08	68.06	55.50	0.002	0.018
XETA TEC:XETA	1047.00	1088.56	1089.10	0.57	0.28	43.80	43.29	-0.004	0.019
XICOR:XICO	800.51	875.07	879.61	0.31	0.60	19.88	35.28	-0.001	0.024
XILINX:XLNX	996.39	1010.63	1010.94	1.24	0.52	56.27	60.35	-0.011	0.022
XOMA LTD:XOMA	889.00	955.59	957.32	0.28	0.08	24.48	17.68	-0.006	0.029
YAHOO!:YHOO	875.05	902.49	903.99	1.12	0.12	39.95	26.90	-0.015	0.027
S&P500:SPD1	1652.32	1691.50	1693.18	0.10	0.15	88.02	90.54	0.002	0.008
Summary Statistics				λ_u	λ_d	η_u	η_d	μ	σ
Mean				0.49	0.30	53.82	48.98	-0.002	0.020
Median				0.40	0.20	49.81	44.51	-0.001	0.019
Std. Dev.				0.30	0.26	24.98	23.76	0.005	0.006
Minimum				0.05	0.02	14.33	14.11	-0.015	0.003
Maximum				1.33	1.06	150.68	162.57	0.008	0.037

Table 3: **Sample statistics for 11 stocks, the S&P500 and NASDAQ Composite.** The daily returns for individual stocks span the period 1/1999 through 12/2003 ($N = 1256$). The raw and dividend adjusted monthly returns for S&P500 (SPM1 and SPM2) span the period 1/1926 through 12/2003 ($N = 936$). The raw and dividend adjusted daily returns for S&P500 (SPD1 and SPD2) span the period 7/1962 through 12/2003 ($N = 10446$). Monthly and daily NASDAQ returns (NASM, $N = 372$ and NASD, $N = 7828$) span the period 1/1973 through 12/2003.

Stock:Ticker	Minimum	Median	Maximum	Mean	SD	Skewness	Kurtosis
ATT CP:T	-0.1908	-0.0022	0.2317	-0.0006	0.0296	0.4847	5.732
CYPRESS BIO:CYPB	-0.2941	0.0000	0.7708	0.0021	0.0724	1.8645	13.889
HYCOR BIO:HYBD	-0.4590	0.0000	0.5852	0.0033	0.0680	1.3077	10.906
INTEL CP:INTC	-0.2203	-0.0004	0.2012	0.0007	0.0360	-0.0916	3.120
LIFECORE BIO:LCMB	-0.5754	0.0000	0.5387	0.0009	0.0499	-0.1043	30.443
MFRI CP:MFRI	-0.2264	0.0000	0.5044	0.0009	0.0534	1.4315	12.678
MONSANTO:MON	-0.1514	0.0000	0.1260	0.0007	0.0267	0.1209	3.042
TIMBERLAND:TBL	-0.1455	0.0000	0.2004	0.0017	0.0298	0.5169	4.323
TIFFANY:TIF	-0.2084	-0.0002	0.2298	0.0015	0.0324	0.4135	4.919
UNITEDHLT:UNH	-0.1952	0.0015	0.1327	0.0016	0.0238	-0.3194	6.170
XICOR:XICO	-0.6361	0.0000	0.5514	0.0039	0.0658	0.3423	11.830
S&P-500:SPM1	-0.2994	0.0090	0.4222	0.0064	0.0563	0.3470	9.291
S&P-500:SPM2	-0.2871	0.0128	0.4168	0.0098	0.0567	0.4350	9.516
S&P-500:SPD1	-0.2047	0.0004	0.0910	0.0003	0.0095	-0.9448	25.758
S&P-500:SPD2	-0.1957	0.0005	0.0886	0.0005	0.0095	-0.8327	22.119
NASDAQ:NASM	-0.2711	0.0152	0.2198	0.0108	0.0663	-0.4899	1.684
NASDAQ:NASD	-0.1132	0.0011	0.1427	0.0005	0.0124	-0.0823	10.778

Table 4: **Comparison of alternative models.** The table contains the BIC values for nine alternative specifications. Bold face shows the minimum BIC value and indcates that the specification fits the data better than all other alternatives. The ***** indicates that the model is the best in the jump diffusion class and ****** means the model is the best among the ARCH specifications.

Ticker	GBM	LJD	DEJD	ARCH(1)	ARCH(2)	GARCH(1,1)	EARCH(1)	EARCH(2)	EGARCH(1,1)
T	-5261.65	-5405.34	**-5406.17**	-5272.57	-5292.83	-5319.06	-5261.31	-5288.62	-5351.33 **
CYPB	-3018.41	**-3409.86**	-3408.85	-3099.79	-3093.39	-3217.97	-3101.18	-3101.73	-3236.06 **
HYBD	-3176.25	-3557.46	**-3564.59**	-3241.99	-3259.72	-3317.69	-3241.52	-3248.18	-3348.98 **
INTC	-4769.75	-4833.66	-4835.93 *	-4792.47	-4823.69	-4895.44	-4781.70	-4806.55	**-4938.38**
LCMB	-3951.71	-4526.84	**-4544.81**	-3955.62	-3996.58	-3967.59	-3946.77	-4009.96 **	-3982.07
MFRI	-3782.55	-4302.66	**-4476.41**	-3993.82	-4029.71	-4045.87 **	-3955.92	-3981.91	-3990.37
MON	-5523.13	-5634.04	**-5640.11**	-5592.01	-5588.83	-5608.79	-5580.71	-5577.64	-5625.30 **
TBL	-5247.67	**-5393.48**	-5392.69	-5241.01	-5246.05	-5273.06	-5235.68	-5242.14	-5284.41 **
TIF	-5035.51	-5172.60 *	-5172.03	-5069.09	-5114.21	-5154.63	-5058.19	-5114.33	**-5198.59**
UNH	-5813.31	-5955.38 *	-5952.89	-5840.13	-5875.06	-5972.17	-5825.02	-5858.03	**-5986.28**
XICO	-3257.33	-3481.52	**-3487.33**	-3279.10	-3289.27	-3370.58	-3271.80	-3290.56	-3382.80 **
SPM1	-2718.08	-2922.65	-2926.23 *	-2780.93	-2906.85	**-3027.71**	-2763.22	-2828.40	-3025.37
SPM2	-2702.04	-2909.73	-2913.85 *	-2770.73	-2900.53	**-3014.57**	-2753.41	-2816.44	-3013.34
SPD1	-67724.75	-69176.61	-69599.32 *	-68799.39	-69351.92	-71047.80	-68569.82	-69067.87	**-71290.30**
SPD2	-67683.35	-69194.25	-69521.58 *	-68702.25	-69268.87	-70966.64	-68482.58	-68998.21	**-71204.70**
NASM	-952.00	-959.13 *	-953.69	-973.78	-979.10	**-990.45**	-967.00	-970.51	-987.12
NASD	-46530.49	-48698.85	-49402.71 *	-48497.41	-49966.14	-51522.57	-47817.94	-48957.51	**-51599.64**

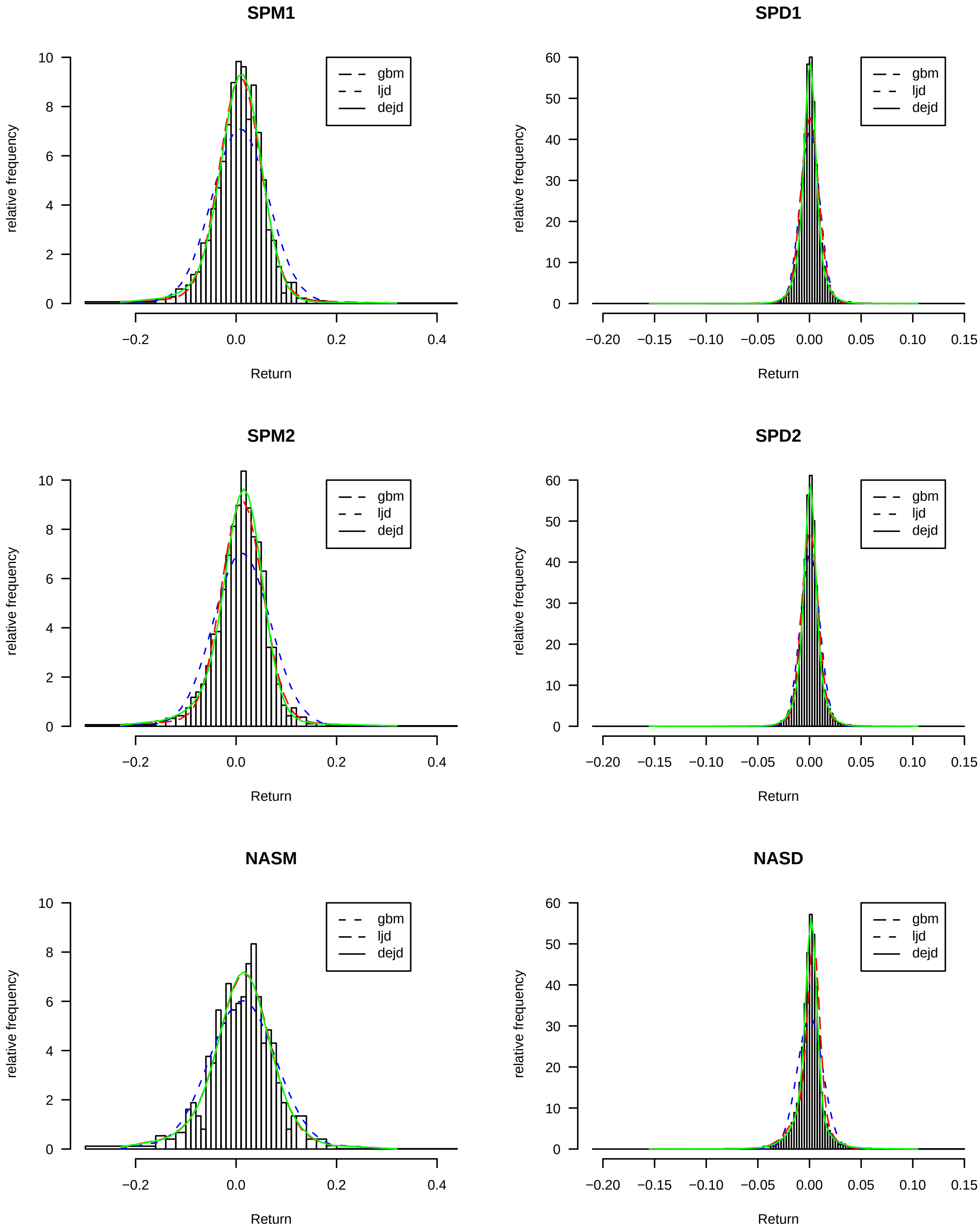
Table 5: **Maximum Likelihood Estimates for GBM, LJD, and DEJD.** Standard Errors appear below the estimates. Bold face indicates that the parameter estimate is significant at 95% or higher confidence level. The symbol **"****"** indicates that DEJD provides the best fit among all alterntaive specifications by the BIC criterion (See Table 4). Similarly, **"***"** indicates LJD is the best, **"**"** indicates that DEJD is better than LJD, and finally **"*"** indicates that LJD is better than DEJD. Dividend and split adjusted daily returns for the 11 stocks span the period 1/1999 through /12/2003 ($N = 1256$). The raw and dividend adjusted monthly returns for S&P500 (SPM1 and SPM2) span the period 1/1926 through 12/2003 ($N = 936$). The raw and dividend adjusted daily returns for S&P500 (SPD1 and SPD2) span the period 7/1962 through 12/2003 ($N = 10446$). Monthly and daily NASDAQ returns (NASM, $N = 372$ and NASD, $N = 7828$) span the period 1/1973 through 12/2003.

	The GBM		Lognormal Jump-Diffusion (LJD)					Double Exponential Jump-Diffusion (DEJD)					
Ticker	μ	σ	λ	α	β	μ	σ	λ_u	λ_d	η_u	η_d	μ	σ
T	1.23E-4	**2.96E-2**	**1.61E-1**	**1.33E-2**	**4.61E-2**	**-2.47E-3**	**2.22E-2**	**3.48E-1**	4.94E-2	**4.74E+1**	**2.90E+1**	**-5.97E-3**	**2.10E-2**
****	8.36E-4	5.91E-4	3.29E-2	4.43E-3	2.85E-3	8.01E-4	6.11E-4	1.20E-1	1.23E-1	2.43	2.15	1.14E-3	7.65E-7
CYPB	**4.70E-3**	**7.24E-2**	**4.47E-1**	**2.53E-2**	**8.72E-2**	**-8.34E-3**	**3.38E-2**	**4.34E-1**	3.54E-1	**1.71E+1**	**2.58E+1**	**-9.16E-3**	**3.09E-2**
***	2.04E-3	1.44E-3	4.23E-2	4.63E-3	3.55E-3	1.62E-3	6.19E-4	1.64E-1	1.97E-1	1.87	1.82	2.24E-3	2.67E-6
HYBD	**5.58E-3**	**6.79E-2**	**6.76E-1**	**1.22E-2**	**7.10E-2**	**-4.66E-3**	**2.56E-2**	**4.66E-1**	**7.22E-1**	**1.94E+1**	**3.33E+1**	**1.29E-3**	**2.30E-2**
****	1.92E-3	1.36E-3	5.17E-2	2.95E-3	2.26E-3	1.46E-3	1.22E-3	2.37E-1	3.37E-1	4.23E-1	4.06E-1	3.16E-4	1.01E-6
INTC	1.37E-3	**3.60E-2**	**5.27E-2**	1.99E-3	**7.54E-2**	1.14E-3	**3.23E-2**	**3.39E-1**	6.10E-2	**4.72E+1**	**2.45E+1**	-3.58E-3	**2.81E-2**
**	1.02E-3	7.19E-4	2.98E-2	1.33E-2	9.43E-3	1.05E-3	7.64E-4	1.59E-1	1.54E-1	1.88	1.70	1.66E-3	2.46E-6
LCBM	2.14E-4	**4.99E-2**	**2.16E-1**	**1.60E-2**	**8.24E-2**	**-2.22E-3**	**2.61E-2**	**3.02E-1**	1.26E-1	**2.42E+1**	**2.06E+1**	**-5.12E-3**	**2.34E-2**
****	1.41E-4	9.96E-4	3.40E-2	5.29E-3	3.09E-3	9.64E-4	7.17E-4	6.13E-2	7.35E-2	1.77	1.56	1.04E-3	6.58E-7
MFRI	2.28E-3	**5.34E-2**	**4.83E-1**	1.32E-3	**6.76E-2**	5.12E-4	**2.06E-2**	**7.18E-1**	**7.52E-1**	**3.31E+1**	**3.57E+1**	**2.43E-4**	**3.85E-3**
****	1.51E-3	1.07E-3	4.34E-2	3.23E-3	2.42E-3	1.02E-3	7.75E-4	1.56E-1	1.88E-1	1.14E-1	1.55E-1	2.25E-6	2.22E-9
MON	1.09E-3	**2.67E-2**	**5.51E-1**	**3.57E-3**	**2.87E-2**	-1.11E-3	**1.56E-2**	**5.37E-1**	2.93E-1	**6.14E+1**	**5.62E+1**	-2.69E-3	**1.55E-2**
****	7.53E-4	5.33E-4	4.83E-2	1.70E-3	1.27E-3	8.55E-3	6.78E-4	2.54E-1	2.74E-1	5.35	4.76	1.54E-3	3.27E-7
TBL	**2.09E-3**	**2.98E-2**	**3.17E-1**	**8.03E-3**	**3.83E-2**	-6.99E-4	**1.97E-2**	**5.03E-1**	3.22E-1	**5.17E+1**	**5.65E+1**	-2.26E-3	**1.72E-2**
***	8.41E-4	5.94E-4	3.83E-2	2.78E-3	2.00E-3	8.61E-4	6.62E-4	2.34E-1	2.64E-1	4.31	3.94	1.54E-3	3.72E-7
TIF	**2.06E-3**	**3.24E-2**	**2.34E-1**	**1.42E-2**	**4.37E-2**	-1.51E-3	**2.32E-2**	**2.99E-1**	3.30E-2	**4.00E+1**	**2.46E+1**	**-4.30E-3**	**2.38E-2**
*	9.15E-4	6.47E-4	3.54E-2	3.71E-3	2.39E-3	9.21E-4	7.05E-4	1.34E-1	1.34E-1	1.94	1.79	1.17E-3	1.47E-6
UNH	1.91E-3	**2.38E-2**	**2.04E-1**	-6.26E-3	**3.55E-2**	**1.91E-3**	**1.72E-2**	**4.01E-1**	3.28E-1	**6.98E+1**	**6.19E+1**	1.30E-3	**1.47E-2**
*	6.71E-3	4.75E-4	3.43E-2	3.05E-3	2.05E-3	6.65E-4	5.00E-4	1.53E-1	1.68E-1	3.90	3.81	1.02E-3	2.29E-7
XICO	**6.05E-3**	**6.58E-2**	**2.25E-1**	**3.98E-2**	**8.91E-2**	**-4.05E-3**	**4.46E-2**	**3.07E-1**	1.32E-2	**1.86E+1**	**8.37**	**-9.98E-3**	**4.56E-2**
****	1.86E-3	1.31E-3	3.50E-2	7.16E-3	3.87E-3	1.71E-3	1.30E-3	5.90E-2	4.73E-2	9.20E-1	1.30	2.07E-3	8.82E-6
SPM1	**7.97E-3**	**5.62E-2**	**8.89E-2**	-2.26E-2	**1.28E-1**	**9.23E-3**	**4.08E-2**	2.94E-2	**1.92E-1**	**9.31**	**1.96E+1**	**1.37E-2**	**3.86E-2**
**	1.84E-3	1.30E-3	3.55E-2	1.88E-2	1.13E-2	1.54E-3	5.77E-3	5.97E-2	7.98E-2	1.82	1.19	1.98E-3	1.00E-5
SPM2	**1.14E-2**	**5.67E-2**	**9.32E-2**	-2.13E-2	**1.27E-1**	**1.27E-2**	**4.08E-2**	4.99E-2	**3.38E-1**	**1.16E+1**	**2.44E+1**	**2.00E-2**	**3.53E-2**
**	1.86E-3	1.31E-3	3.56E-2	1.84E-2	1.10E-2	1.55E-3	2.80E-3	5.18E-2	8.29E-2	1.61	1.64	2.25E-3	5.12E-6
SPD1	**3.76E-4**	**9.45E-3**	**4.22E-2**	8.29E-4	**2.37E-2**	**3.26E-4**	**8.54E-3**	**4.64E-1**	**5.62E-1**	**1.74E+2**	**1.86E+2**	**7.01E-4**	**4.67E-3**
**	9.25E-5	6.54E-5	1.03E-2	1.42E-3	7.97E-4	9.86E-5	6.67E-5	7.14E-2	9.33E-2	4.30E-1	4.44E-1	1.23E-5	1.65E-9
SPD2	**5.12E-4**	**9.47E-3**	**7.37E-2**	5.36E-4	**1.99E-2**	**4.26E-4**	**8.00E-3**	**4.72E-1**	**5.72E-1**	**1.74E+2**	**1.86E+2**	**8.39E-4**	**4.63E-3**
**	9.27E-5	6.55E-5	1.06E-2	9.16E-4	5.39E-4	9.51E-5	6.58E-5	7.11E-2	9.22E-2	3.65E-1	3.84E-1	1.05E-5	1.86E-9
NASM	**1.30E-2**	**6.62E-2**	**3.55E-1**	-1.93E-2	**7.47E-2**	**1.88E-2**	**4.77E-2**	7.63E-1	4.41E-1	**4.00E+1**	**2.28E+1**	1.20E-2	**4.21E-2**
*	3.44E-3	2.43E-3	7.37E-2	1.24E-2	1.01E-2	4.39E-3	3.20E-3	8.92E-1	1.05	7.08	6.32	1.71E-2	4.69E-5
NASD	**5.62E-3**	**1.24E-2**	**5.52E-1**	-4.92E-3	**1.37E-2**	**4.07E-3**	**5.66E-3**	**2.30E-1**	**4.37E-1**	**9.59E+1**	**1.10E+2**	**2.06E-3**	**5.04E-3**
**	1.40E-3	9.89E-5	1.96E-2	2.66E-4	1.70E-4	1.21E-4	1.04E-4	2.38E-2	3.52E-2	6.05E-1	6.99E-1	2.00E-5	2.63E-9

Table 6: **Parameter estimates for monthly S&P500.** The table reports parameter estimates for GBM, LJD, and DEJD specifications for sub-periods of 1/1926 through 12/2003. The data are monthly returns for the S&P500 (SPM1).

		The GBM		Lognormal Jump-Diffusion (LJD)					Double Exponential Jump-Diffusion (DEJD)					
Period	n	μ	σ	λ	α	β	μ	σ	λ_u	λ_d	η_u	η_d	μ	σ
01/26–12/55	360	3.597E-3	0.0728	0.178	-0.042	0.133	0.0120	0.044	0.056	0.411	9.40	17.68	0.0214	0.0386
12/55–12/75	240	2.852E-3	0.0399	1.574	-0.012	0.026	0.0221	0.017	0.052	1.235	22.50	50.62	0.0253	0.0231
12/75–12/95	240	8.005E-3	0.0422	0.446	-0.006	0.046	0.0168	0.027	0.395	0.457	46.51	38.18	0.0117	0.0272
01/26–12/95	840	4.644E-3	0.0569	0.115	-0.034	0.119	0.0093	0.039	0.065	0.222	14.53	18.24	0.0131	0.0353
01/26–12/03	936	7.970E-3	0.0562	0.089	-0.023	0.128	0.0092	0.040	0.029	0.192	9.31	19.60	0.0137	0.0386

Figure 1: Histograms of returns and fitted models



Notes

[1] For a comprehensive overview of recent developments in modeling equity price processes see the collection of papers and their references in the special issue of the Journal of Econometrics (Vol. 116, 2003) entitled “Frontiers of Financial Econometrics and Financial Engineering”.

[2] Bakshi & Cao (2004) refer to their specification as *double jump* model. They find that this specification does a better job of explaining the tail-size and asymmetry for individual stock returns. More importantly, they find that jump in prices, rather than the volatility, is more relevant for stock option prices. Eraker et al. (2003) focus on the S&P 500 and NASDAQ-100 index returns. They report strong evidence supporting jumps in both the price and volatility. They show that jump in volatility increases the implied volatility for out-of-the-money options. Moreover, they show that parameter uncertainty can explain as much as 2% of the Implied volatility.

[3]Wu (forthcoming) recently proposed a pure jump-Lévy process that also permits for a distinction between up and down jumps in security prices.

[4] Ramezani & Zeng (1998) propose their model from an econometric viewpoint. These papers are clearly complementary: While Ramezani and Zeng (1998) focus on the problem of parameter estimation, Kou (2002) and Kou & Wang (2004) develop the DEJD option pricing formula, which would require the estimated parameters as inputs.

[5] The literature based on DEJD is large and expanding. For example, Huang & Huang (2003) study the link between corporate-Treasury Yield spread and default risk by calibrating a variety of specifications for the firm equity value, including DEJD. Metayer (2003) undertake a more detailed study of these issues by specifically focusing on the DEJD. Carr & Wu (2004) and Huang & Wu (2004) show that the symmetrical form of the DEJD is a special case of the general time-changed Lévy process. They use the characteristic function methodology to price contingent claims via an efficient Fast Fourier Transform (FFT) technique. This methodology enable them to select and test alternative option pricing models. Lewis (2001), Broadie & Yamamoto (2003), Cont & Tankov (2004) and Dupoyet (2004) also use variations of the FFT method to develop an option pricing formula for DEJD specification. dH́alluin, Forsyth & Vetzal (2004) develop robust numerical methods for pricing contingent claims under jump diffusion processes, with particular emphasis on DEJD. Using Laplace Transform, Sepp (2004) develops analytical pricing formulas for barrier options under DEJD. Lee (forthcoming) undertakes a systematic study of the determinant of the shape of the implied volatility surface, establishing how the shape of this surface changes with strike and time to maturity for a host of processes, including the DEJD. He shows that the larger the expected size of an up jump, the fatter the right-hand tail of the spot price distribution at maturity (the intuition for the down jump is similar). He claims that the jump frequency has no effect on the asymptotic slope of the implied volatility (under the risk neutral measure). Cont & Tankov (2004) propose a non-parametric method for calibrating jump-diffusion option pricing models, including the DEJD specification. Using simulations studies, these authors show that their proposed method can closely approximate the assumed parameters for the underlying process. Keppo et al. (2003) extend Kou (2002) by appending stochastic volatility to DEJD. They find that down jumps

effects S&P 500 options more than up jumps or stochastic volatility.

[6] Using daily data, Ramezani and Zeng (1998) provided estimates for 6 firms and the S&P-500 index for the period 1991-92. However, they did not undertake a comprehensive examination and comparison, as is done here.

[7] Ekholm & Pasternack (2002) link the negative skewness in returns to management's strategic disclosure of news about the firm's prospects. They show that returns for days with non-scheduled news arrival are negatively skewed.

[8] Maheu & McCurdy (2004) distinguish between *normal* and *unusual* news events, which have different impacts on returns and volatility. They consider the impact of jumps within the GARCH specification.

[9] A Lévy process can be decomposed as the sum of three independent components: A linear drift, a Brownian motion and a pure jump process. For recent application of Lévy process in finance see Cont & Tankov (2003), Carr & Wu (2004) and Huang & Wu (2004) and their references.

[10] A Fortran program for pricing European options for this special case of the DEJD is available from Professor Tsay's web page: `http://www.gsb.uchicago.edu/fac/ruey.tsay/teaching/fts/kou.f`. Mathematica code for pricing European options under the general DEJD is available from Professor Kou (`sk75@columbia.edu`).

[11] Assuming a range of parameter values, Kou (2002) and others provide evidence that the DEJD can generate the widely documented patterns of volatility smile and smirk. Using Kou's option pricing formula, firm specific data, and the parameter estimates reported below, we arrived at identical conclusions. These findings will not be presented here to save space but are available from authors upon request.

[12] For references on ARCH see Engle (2001), Tsay (2002) and Gourieroux (1997).

[13] Of course these values are based on the *mean* of the parameter estimates. The estimated mean up (down) jump range is 0.66% to 7.50% (-0.61% to -6.66%). The estimated up (down) jump intensity range is 1.5 to 21.5 (1 to 51) days.

[14] No dividend adjusted series are available since few firms on NASDAQ pay dividends.

References

Aït-Sahalia, Y. (2002), 'Maximum-likelihood estimation of discretely-sampled diffusions: A closed-form approximation approach', *Econometrica* **70**, 223–262.

Aït-Sahalia, Y. (forthcoming), 'Disentangling diffusion from jumps', *Journal of Financial Economics*.

Aït-Sahalia, Y. & Hansen, L. P. (2004), *Handbook of Financial Econometrics*, Amsterdam:North-Olland.

Andersen, L. & Andreasen, J. (2000), 'Jump-diffusion processess: Volatility smile fitting and numerical methods for pricing', *Review of Derivative Research* **4**, 231–262.

Andersen, T. G., Benzoni, L. & Lund, J. (2002), 'An empirical investigation of continuous-time equity return models', *Journal of Finance* **57**, 1239–1284.

Bakshi, G. & Cao, C. (2004), 'Disentangling the contribution of return-jumps and volatility-jumps on individual equity option prices', *Working Paper, Smith Business School, University of Maryland*.

Bakshi, G., Cao, C. & Chen, Z. (1997), 'Empirical performance of alternative option pricing models', *The Journal of Finance* **52**(5), 2003–2049.

Bates, D. S. (2003*a*), 'Empirical option pricing: A retrospection', *Journal of Econometrics* **116**(1-2), 387–404.

Bates, D. S. (2003*b*), 'Maximum likelihood estimation of latent affine processes', *Working Paper, University of Iowa*.

Broadie, M. & Yamamoto, Y. (2003), 'Application of the fast gauss transform to option pricing', *Management Science* **49**(8), 1071–1088.

Carr, P. & Wu, L. (2004), 'Time-changed Lévy processess and option pricing', *Journal of Financial Economics* **17**(1), 113–141.

Chernov, M., Gallant, A. R., Ghysels, E. & Tauchen, G. (2003), 'Alternative models for stock price dynamics', *Journal of Econometrics* **116**(1-2), 225–257.

Cont, R. & Tankov, P. (2003), *Financial modelling with Jump Processes*, Chapman & Hall / CRC, Financial Mathematic Series.

Cont, R. & Tankov, P. (2004), 'Nonparametric calibration of jump-diffusion option pricing models', *Journal of Computational Finance* **7**(3), 1–49.

dH́alluin, Y., Forsyth, P. & Vetzal, K. (2004), 'Robust numerical methods for contingent claims under jump diffusion processes', *Working Paper, School of Computer Science, University of Waterloo*.

Duffie, D., Pan, J. & Singleton, K. J. (2000), 'Transform analysis and asset pricing for affine jump-diffusions', *Econometrica* **68**(6), 1343–1376.

Dupoyet, B. (2004), 'Asymmetric jump processess: Option pricing implications', *Working Paper*.

Ekholm, A. & Pasternack, D. (2002), 'The negative news threshold - an explanation for negative skewness in stock returns', *Working Paper 465, Swedish School of Economics and Business Administration.*

Engle, R. F. (2001), 'GARCH 101: The use of ARCH/GARCH models in applied econometrics', *Journal of Economic Perspectives* **15**(4), 157–168.

Eraker, B., Johannes, M. & Polson, N. (2003), 'The impact of jumps in volatility and returns', *Journal of Finance* **58**(3), 1269–1300.

Garcia, R., Ghysels, e. & Renault, E. (2004), 'The econometrics of option pricing', *Working Paper, Universit de Montral - CIREQ / Dpartement de sciences conomiques*.

Gourieroux, C. (1997), *ARCH Models and Financial Applications*, Springer-Verlag.

Hamilton, J. D. (1994), *Time Series Analysis*, Princeton University Press.

Honoré, P. (1998), 'Pitfalls in estimating jump diffusion models', *Working Paper, University of Aarhus.*

Huang, J. & Huang, M. (2003), 'How much of the corporate-treasury yield spread is due to credit risk?', *Working Paper, Graduate School of Business, Stanford University*.

Huang, J. & Wu, L. (2004), 'Specification analysis of option pricing models based on time-changed Lévy processes', *Journal of Finance* **59**(3), 1405–1439.

Jackwerth, J. & Rubinstein, M. (1996), 'Recovering probability distributions from option prices', *Journal of Finance* **51**, 1611–1631.

Keppo, J., Meng, X., Shive, S. & Sullivan, M. (2003), 'Modelling and hedging options under stochastic pricing parameters', *Working Paper, Industrial and Operations Engineering, University of Michigan at Ann Arbor*.

Kiefer, N. M. (1978), 'Discrete parameter variation: Efficient estimation of switching regression model', *Econometrica* **46**(2), 427–34.

Kou, S. (2002), 'A jump diffusion model for option pricing', *Management Science* **48**(8), 1086–1101.

Kou, S. & Wang, H. (2004), 'Option pricing under a double exponential jump diffusion model', *Management Science* **50**(9), 1178–1192.

Lee, R. W. (forthcoming), 'Implied volatility: Statics, dynamics, and probabilistic interpretation', *Recent Advances in Applied Probability*.

Lewis, A. (2001), 'A simple option formula for general jump-diffusion and other exponential Lévy processes', *Envision Financial System and OptionCity.net*.

Maheu, J. M. & McCurdy, T. (2004), 'News arrival, jump dynamics and volatility components for individual stock returns', *The Journal of Finance* **59**(2), 755–768.

Merton, R. C. (1976a), 'Option pricing when underlying stock returns are discontinuous', *Journal of Financial Economics* **3**, 224–244.

Merton, R. C. (1976b), ‘The impact on option pricing of specification error in the underlying stock price returns’, *The Journal of Finance* **31**(2), 333–350.

Metayer, B. (2003), ‘A double exponential jump diffusion process to modelling risky bond prices’, *Working Paper, Swiss Banking Institute, University of Zurich*.

Milgrom, P. R. (1981), ‘Good news and bad news: Representation theorems and applications’, *Bell Journal of Economics* **12**, 380–91.

Naik, V. (1993), ‘Option valuation and hedging strategies with jumps in the volatility of asset returns’, *The Journal of Finance* **48**(5), 1969–1984.

Patel, J., Kapadia, C. H. & Owen, D. B. (1976), *Handbook of Statistical Distributions*, Marcel Dekker Inc.

Press, S. J. (1967), ‘A compound events model for security prices’, *Journal of Business* **40**, 317–335.

Press, W. H., Teukolsky, S. A., Vetterling, W. T. & Flannery, B. P. (1992), *Numerical Recipes in FORTRAN*, Cambridge University Press.

Protter, P. (1991), *Stochastic integration and differential equations*, Springer-Verlag, New York.

Ramezani, C. & Zeng, Y. (1998), ‘Maximum likelihood estimation of asymmetric jump-diffusion processes: Application to security prices’, *Working Paper, Department of Mathematics and Statistics, University of Missouri, Kansas City*.

Schwarz, G. (1978), ‘Estimating the dimension of a model’, *Annals of Statistics* **6**, 461–464.

Sepp, A. (2004), ‘Analytical pricing of double-barrier options under a double-exponential jump diffusion process: Applications of laplace transform’, *International Journal of Theoretical and Applied Finance* **7**(2), 151–175.

Sorensen, M. (1991), Likelihood methods for diffusions with jumps, *in* N. V. Prabhu & I. V. Basawa, eds, ‘Statistical Inference in Stochastic Processes’, Marcel Dekker, Inc., New York, USA.

Sundaresan, S. M. (2000), ‘Continuous-time methods in finance: A review and an assessment’, *Journal of Finance* **55**(4), 1569–1622.

Tsay, R. S. (2002), *Analysis of Financial Time Series*, Wiley Series in Probability and Statistics, Wiley Inter-Science.

Wu, L. (2003), ‘Jumps and dynamic asset allocation’, *Review of Quantitative Finance and Accounting* **20**(3), 207–243.

Wu, L. (forthcoming), ‘Dampened power law: Reconciling the tail behavior of financial security returns’, *Journal of Business*.