# BEHAVIORAL STATISTICAL ARBITRAGE*

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# ABSTRACT

One of the inefficiencies observed on the financial markets is a momentum effect. This inefficiency can be exploited by a trading strategy. Most of the empirical studies of momentum effect were made on the US stock market. In this thesis we test the momentum effect on the European markets, in particular, on the Swiss, French and German and elaborate a portfolio optimisation strategy, which would enable us to realise positive returns on the momentum portfolios.

To implement this we use cumulative returns as an indicator of “winners” and “losers” stocks to be included into the portfolio and develop three approaches to portfolio optimisation: minimisation of variance of the portfolio, minimisation of covariance between long and short positions in the portfolio and minimisation of variance and covariance of the portfolio while holding beta of the portfolio equal 0. We also test two measurement periods: 6-month and 1-year and three holding periods: 1-month, 4-month and 6-month.

The obtained results prove, that the strategy can generate positive returns, but there is no common strategy for all markets studied, which can be explained by national specifics, different number of market participants, number of stocks available, etc.

The main achievement of this thesis is the elaboration of portfolio optimisation models for implementation of behavioural statistical arbitrage strategy under the certain investments constraints, which allows us to obtain the targeted risk/return profile of the portfolio.

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# 1. Introduction

## 1.1. Objective

As empirical evidence shows, financial markets demonstrate some inefficiencies, which can hardly be explained by traditional finance. One of those inefficiencies is a momentum effect. Under momentum effect stock prices, which were growing for some time in the past (from 6 months to 1 year) continue to rise even further over their fundamental value for another several months instead of falling to their fundamental value under the influence of rational investors trying to exploit the arbitrage opportunity.

Most of empirical studies on momentum effect were made on the US stock markets. The objective of this Master Thesis is to test the momentum effect on the European markets, in particular on constituents of the Swiss, French and German market indices and to elaborate portfolio optimisation models to implement statistical arbitrage. These market indices were chosen because they include small numbers of stocks, which make the calculations easier and less time-consuming. However the models can easily be extended to a larger number of stocks.

## 1.2. Methodology

The data used in our paper includes mid-week closing dividend and splits adjusted price data taken from the period of 02.01.1985 - 09.07.2003 for the Swiss and French markets, and of 03.07.1991 - 09.07.2003 for the German market.

To exploit the momentum effect first we choose “winners” and “loosers” among the available stocks on the basis on their cumulative return, which was proved to be the most important variable in seeking the momentum effect. There may be other ways of ranking the stocks, but taking into account the small number of stocks available, we don’t consider it appropriate to test them.

To get the better view of the duration of momentum effect on the chosen markets we take two measurement periods – 6 months and 1 year, and three holding periods – 1 month, 4 months and 6 months.

The second stage is to form a portfolio and elaborate the optimisation model. We form a portfolio as a combination of two sub-portfolios: one is long on 5 “winners” stocks; another is short on 5 “loosers” stocks. We put the weight constraints for the stocks in the sub-portfolios to be minimum

10% maximum 60%. The important condition is also zero cost of the strategy, i.e. sub-portfolios should sum up to 0.

To solve the portfolio optimisation problem under the investment constraints, we use three models:

1. Portfolio variance minimization,
2. Covariance minimization between sub-portfolios,
3. Minimization of portfolio variance and covariance between long and short portfolios under zero-beta condition. For this case we take only 4 months holding period and both 6-month and 1-year measurement periods.

## 1.3. Conclusions

The main achievement of this thesis is the elaboration of portfolio optimisation models for implementation of behavioural statistical arbitrage strategy under the certain investments constraints, which allows us to obtain the targeted risk/return profile of the portfolio.

The implemented models have proved, that it is possible to outperform the market using the strategy proposed. On the Swiss market the strategy generates the highest positive returns with comparison to the market index and it outperforms the market in the largest number of cases. On the German market the strategy demonstrates the worst performance with the smallest number of positive results.

In terms of measurement and holding period the best performing strategy on the Swiss market corresponds to the classical momentum with a measurement period of 6 months and holding period of 4 months. For French and German markets the better measurement period is equal to 1 year.

The best performing strategy for all markets is the zero-beta strategy, which is implemented on the basis of 6-month measurement period and 1-year measurement period for the Swiss, French and German markets.

Taking into account all mentioned above, we can make a conclusion, that there is no common model that can be applied on all of the chosen markets. This can be explained by national specifics of the markets, number of active participants on the markets and stocks available.

## 1.4. Outline

In the second part of our thesis we give the overview of behavioural finance as an alternative to traditional paradigm. We explain the limits to efficient market hypothesis and some psychological issues, which lie in the basis of behavioural theories. We also give here an overview of some theories and approaches developed in the literature to the financial markets' phenomena observed on the aggregate stock market, cross-section of average returns, and fund comovement. At the end of part one we characterise such behavioural trading strategies as momentum and contrarian strategies and the interplay between the two.

The third part of the paper is devoted to the overview of the hedge fund industry, its role and strategies, used by hedge funds. Then we concentrate more on the statistical arbitrage strategy, assumptions, which underlie the strategy and give some examples of statistical arbitrage trading models.

The fourth part is the empirical part of the thesis. It combines the behavioural aspect and statistical arbitrage approach. It contains explanations on the data used, methodology and illustrates the portfolio optimisation methods. Here we also present the results obtained from portfolio simulations.

The last part of the paper contains conclusions and results, which we obtained from our simulations.

## 2. Behavioural Finance As A New Approach To Financial Markets

### 2.1. Overview Of Behavioural Finance

The traditional finance paradigm seeks to understand financial markets using models in which agent are "rational", which means:

1. When agents receive new information they update their beliefs correctly.
2. Given their beliefs, agents make choices consistently.

However, sometimes financial markets demonstrate behaviour, which can hardly be explained by traditional finance. Among such financial phenomena we could mention the behaviour of the aggregate stock market and cross-section of average returns.

Behavioural finance is a new approach to financial markets, which argues, that some of those phenomena can be better understood using models, in which some agents are not fully rational. Different theories of behavioural finance rely on releasing of one or both constraints of rationality. We will give an overview of some behavioural theories and their applications to the mentioned phenomena later in this part.

Behavioural finance consists of 2 building blocks:

1. Limits to arbitrage – includes theoretical studies, which show that irrationality can have a substantial and long-lived impact on prices and rational investors cannot always undo this impact through arbitrage.
2. Psychology – behavioural models often need to specify the form of agents' irrationality and define how people form their beliefs and preferences.

#### 2.1.1. Market Efficiency And Limits To Arbitrage

Efficient Markets Hypothesis (EMH) states, that a security's price reflects its "fundamental value", i.e. the sum of discounted expected cash flows, where in forming expectations investors correctly process all available information and where the discount rate is consistent with a normatively acceptable preference specification. *In efficient market no investment strategy can earn excess risk-adjusted average returns.*

The traditional approach states, that even though irrational traders, known as "noise traders" can influence the price in the short run, rational traders, known as "arbitragers", will immediately exploit the attractive investment opportunity and implement the arbitrage strategy thereby correcting the mispricing.

Behavioural finance argues that implementation of such strategy can often be both risky and costly, thereby allowing the mispricing to survive. Some of the risks, faced by arbitragers, are:

1. Fundamental risk. After arbitrager's exploiting of security underpricing, a piece of bad news about the company can course the price to fall even further. As long as it's very difficult to find a perfect substitute for an individual stock, fundamental risk plays an important role in implementation of arbitrage strategy.
2. Noise trader risk. If pessimism of irrational investors could course underpricing of security, they can become even more pessimistic, pushing the price even lower. This may course losses if arbitrager has short horizon and is not able to wait till the price will finally normalize.
3. Short-sales constraints (fees and legal constraints).
4. Cost of finding and learning about the mispricing.
5. Cost of resources needed to exploit it.

Taking into account the mentioned constraints on arbitrage, we can conclude that mispricing on the market is not necessarily eliminated immediately and may take place for quite a long period of time.

One of strong evidence of long-lasting mispricing is index inclusion. It was noticed, that after inclusion into the S&P 500, a stock jumps in price by an average of 3.5% and much of this jump is permanent. Meanwhile, its fundamental value doesn't change and Standard and Poor's emphasizes that in selecting stocks for inclusion, they are simply trying to make their index representative of the US economy, not to convey any information about the level of riskiness of a firm's future cash flows.

### 2.1.2.Psychology

The theory of limited arbitrage shows that if irrational traders cause deviations from fundamental value, rational traders will often be powerless to do anything about it. In order to say more about the structure of these deviations, behavioural models often assume a specific form of irrationality. For guidance on this, much research was done on the systematic biases that arise when people form beliefs, and preferences.

The most significant research on this topic was made by: Camerer (1995) and Rabin (1998), Kahneman, Slovic and Tversky (1982), Kahneman and Tversky (2000) and Gilovich, Griffin and Kahneman (2002). We will not go deep in describing research made, but will summarize the main results.

Beliefs and Preferences

A crucial component of any model of financial markets is a specification of how agents form expectations and make choice between different options. Psychologists found the following results regarding the way, people form their beliefs:

1. Overconfidence. Extensive evidence shows that people are overconfident in their judgments. This appears in *two guises*. First, the confidence intervals people assign to their estimates of quantities are far too narrow. Second, people are poorly calibrated when estimating probabilities: events they think are certain to occur actually occur only around 80 percent of the time, and events they deem impossible occur approximately 20 percent of the time.
2. Optimism and Wishful Thinking. Most people display unrealistically rosy views of their abilities and prospects.
3. Representativeness. Representativeness leads to *sample size neglect bias*. This means that in cases where people do not initially know the data generating process, they will tend to infer it too quickly on the basis of too few data points.
4. Conservatism. People tend to underweight new information relative to prior.
5. Belief Perseverance. There is much evidence that once people have formed an opinion, they cling to it too tightly and for too long. At least two effects appear to be at work. First, people are reluctant to search for evidence that contradicts their beliefs. Second, even if they find such evidence, they treat it with excessive scepticism.

Experimental evidence shows, that when people form their preferences they systematically violate expected utility theory, which goes back to Von Neumann and Morgenstern (1947) and is widely used by traditional finance. We can summarize the following results obtained by researchers regarding the way, people form preferences:

<u>Prospect Theory:</u>

- Certainty effect. People place much more weight on outcomes that are certain relative to outcomes that are merely probable, then they should according to EU approach.
- Framing. Preferences depend on problem description. There are numerous demonstrations of a 30 to 40 percent shift in preferences depending on the wording of a problem.

- Narrow framing. Tendency to treat individual gambles separately from other portions of wealth.

Ambiguity Aversion.

In reality probabilities are rarely objectively known. Experimental results show that people do not like situations where they are uncertain about the probability distribution of a gamble. Such situations are known as situations of ambiguity, and the general dislike for them, as ambiguity aversion. In the real world, ambiguity aversion has much to do with how competent an individual is at assessing the relevant distribution.

### 2.2. Behavioural Approach To Some Financial Phenomena

As it was mentioned above, financial markets demonstrate phenomena, which can hardly be explained by traditional finance. In this part we want to give an overview of behavioural approaches to some of those phenomena.

#### 2.2.1.Aggregate Stock Market

1. Equity Premium Puzzle – historically stock market earned a high excess rate of return.

- Evidence. Using annual data from 1871-1993, Campbell and Cochrane (1999) report that the average log return of the S&P 500 index is 3,9% higher than the average log return on short term commercial paper.

Behavioural approach.

The core of the equity premium puzzle is that even though stocks appear to be an attractive asset - they have high average returns and a low covariance with consumption growth, investors appear very unwilling to hold them and demand a substantial risk premium in order to hold the market supply. To date, behavioural finance has pursued *two* approaches to this puzzle: one relies on prospect theory, the other on ambiguity aversion.

Prospect theory suggests:

1. Investors treat gambles separately. In financial context this means, that people may choose a portfolio allocation by computing for each allocation the potential gains and losses in the value of their holdings, and then take the allocation with the highest prospective utility.

2. Loss aversion of investors depends on the frequency at which information is presented to them. For example, on daily basis, stocks go down in value almost as often as they go up, so for an investor, who calculates gains and losses of a portfolio daily, loss aversion makes stocks appear unattractive. In contrast, loss aversion does not have much effect on investor's perception of stocks if he calculates the return once per decade.

One of the earliest papers to link prospect theory to the equity premium puzzle is Benartzi and Thaler (1995). They study how an investor with prospect theory type preferences allocates his financial wealth between T-Bills and the stock market. They calculated how often investors would have to evaluate their portfolios to make them roughly indifferent, between stocks and bonds. They found the answer to be a year. This result seems natural, as long as all financial and tax statements are prepared on a yearly basis. This, in turn, suggests a simple way of understanding the high historical equity premium. If investors get utility from annual changes in financial wealth and are loss averse over these changes, their fear of a major drop in financial wealth will lead them to demand a high premium as compensation.

Equity puzzle is in large part a consumption puzzle: given the low volatility of consumption growth, why are investors so reluctant to buy a high return asset, stocks, especially when that asset's covariance with consumption growth is so low? Barberis, Huang and Santos (2001) attempt to build prospect theory into a dynamic equilibrium model of stock returns. They show that loss aversion can indeed provide a partial explanation of the high Sharpe ratio on the aggregate stock market.

Both approaches are effectively assuming that investors engage in narrow framing, both cross-sectionally and temporally. Even if they have many forms of wealth, both financial and non-financial, they still get utility from changes in the value of one specific component of their total wealth: financial wealth in the case of BT and stock holdings in the case of BHS. And even if investors have long investment horizons, they still evaluate their portfolio returns on an annual basis.

## Ambiguity Aversion

Ambiguity aversion suggests that people don't like gambles, for which they can't evaluate the probability distribution.

One of the more popular approaches supposes that when faced with ambiguity, people entertain a range of possible probability distributions and act to maximize the minimum expected utility under any candidate distribution. In effect, people behave as if they expect the actual distribution to be such as to leave them as worse off as possible.

Maenhout (1999) applies this framework to the issue of the equity premium. He shows that if investors are concerned that their model of stock returns is misspecified, they will charge a substantially higher equity premium as compensation for the perceived ambiguity in the probability distribution. He notes, however, that to explain the full 3.9% equity premium requires an unreasonably high concern about misspecification. At best then, ambiguity aversion is only a partial resolution of the equity premium puzzle.

2. Volatility Puzzle – stock returns and price-dividend ratios are both highly variable.

- Evidence. In the same data set mentioned above, the annual standard deviation of excess log returns on the S&P 500 is 18%, while the annual standard deviation of the log price-dividend ratio is 27%.

Behavioural approach.

We can group behavioural approaches to the volatility puzzle by whether they focus on beliefs or on preferences:

Beliefs

1. One possible explanation is that investors believe that the mean dividend growth rate is more variable than it actually is. When they see a surge in dividends, they are too quick to believe that the mean dividend growth rate has increased. Their exuberance pushes prices up relative to dividends, adding to the volatility of returns. This is a direct application of *representativeness* and in particular, of the version of representativeness known as the law of small numbers, where people expect even short samples to reflect the properties of the parent population.

2. Another belief-based approach relies more on private, rather than public information, and in particular, on overconfidence about private information. Suppose that an investor has seen public information about the economy, and has formed a prior opinion about future cash-flow growth. He then does some research on his own and becomes overconfident about the information he gathers: he overestimates its accuracy and puts too much weight on it relative to his prior. If the private information is positive, he will push prices up too high relative to current dividends, again adding to return volatility.

These ideas have a lot in common with those explaining *cross-sectional* anomalies, which we will describe in the next section.

Preferences

In explaining volatility puzzle using preferences approach, Barberis, Huang and Santos (2001) appeal to experimental evidence about dynamic aspects of loss aversion. This evidence suggests that the degree of loss aversion is not the same in all circumstances but depends on prior gains and losses. In particular, Thaler and Johnson (1990) find that after prior gains, subjects take on gambles they normally do not, and that after prior losses, they refuse gambles that they normally accept. One interpretation of this evidence is that losses are less painful after prior gains because they are cushioned by those gains. However, after being burned by a painful loss, people may become more wary of additional setbacks.

Suppose that there is some good cash-flow news. This pushes the stock market up, generating prior gains for investors, who are now less scared of stocks: any losses will be cushioned by the accumulated gains. They therefore discount future cash flows at a lower rate, pushing prices up still further relative to current dividends and adding to return volatility.

### 2.2.2. Cross-Section Of Average Returns

Empirical studies about the cross-section of average returns also revealed some anomalies, which can hardly be explained by the most used and intuitive model – Capital Asset Pricing Model.

1. Size Premium.

Using data on returns of stocks traded on NYSE, AMEX, and NASDAQ during the period from 1963 to 1990 Fama and French (1992) found that the average return of the group of stocks, which have smallest market capitalization, is 0.74% per month higher than the average return of the group of stocks with largest market capitalization. This is anomaly relative to CAPM, because while stocks with the smallest market capitalization do have higher betas, the difference in risk is not enough to explain the difference in average returns.

2. Predictive Power of Scaled-Price Ratios

From the same data set, Fama and French group all stocks into deciles based on their book-to-market ratio, and measure the average return of each decile over the next year. They found that the average return of the highest B/M-ratio decile, containing so called "value" stocks, is 1.53% per month higher than the average return on the lowest-B/M-ratio decile, "growth" or "glamour" stocks, a difference much higher than can be explained through differences in beta between the two portfolios.

Repeating the calculations with the earnings-price ratio as the ranking measure produces a difference of 0.68% per month between the two extreme decile portfolios.

3. Long-Term Reversals.

Every three years from 1926 to 1982, De Bondt and Thaler (1985) rank all stocks traded on the NYSE by their prior three year cumulative return and form two portfolios: a "winner" portfolio of the 35 stocks with the best prior record and a "loser" portfolio of the 35 worst performers. They then measure the average return of these two portfolios over the three years subsequent to their formation. They find that over the whole sample period, the average annual return of the loser portfolio is higher than the average return of the winner portfolio by about 8% per year.

4. Momentum Effect

Every month from January 1963 to December 1989, Jegadeesh and Titman (1993) group all stocks traded on the NYSE into deciles based on their prior six month return and compute average returns of each decile over the six months after portfolio formation. They find that the decile of biggest prior winners outperforms the decile of biggest prior losers by an average of 10 percent on an annual basis.

Comparing this result to De Bondt and Thaler's (1985) study of prior winners and losers illustrates the crucial role played by the length of the prior ranking period. In one case, prior winners continue to win; in the other, they perform poorly. A challenge to both behavioural and rational approaches is to explain why extending the formation period switches the result in this way.

5. Event Studies:

Event studies examine how important corporate announcements influence the stock prices.

- Earnings Announcements

Every quarter from 1974 to 1986, Bernard and Thomas (1989) group all stocks traded on the NYSE and AMEX into deciles based on the size of the surprise in their most recent earnings announcement. They found that on average, over the 60 days after the earnings announcement, the decile of stocks with surprisingly good news outperforms the decile with surprisingly bad new by an average of about 4 percent, a phenomenon known as post-earnings announcement drift. A later study by Chan. Jegadeesh and Lakonishok (1996) measures surprise in other ways relative to analyst expectations, and by the stock price reaction to the news and obtains similar results.

- Dividend Initiations and Ommissions

Michaely, Thaler and Womack (1995) study firms, which announced initiation or omission of a dividend payment between 1964 and 1988. They found, that on average, the shares of firms initiating (omitting) dividends significantly outperform (underperform) the market portfolio over the year after the announcement.

- Stock Repurchases

Ikenberry, Lakonishok and Vermaelen (1995) look at firms, which announced a share repurchase between 1980 and 1990, while Mitchell and Stafford (2001) study firms which did either self-tenders or share repurchases between 1960 and 1993. The latter study finds that on average, the shares of these firms outperform a control group matched on size and book-to-market market by a substantial margin over the four-year period following the event.

- Primary and Secondary Offerings

Loughran and Ritter (1995) study firms, which undertook primary or secondary equity offerings between 1970 and 1990. They find that the average return of shares of these firms over the five-year period after the issuance is markedly below the average return of shares of non-issuing firms matched to the issuing firms on size.

<u>Belief-based behavioral models:</u>

1. Representativeness and Conservatism. Barberis, Shieifer and Vishny (1998), argue that much of the above evidence is the result of systematic errors that investors make when they use public information to form expectations of future cash flows. They build a model that incorporates two of the updating biases: conservatism, the tendency to underweight new information relative to priors, and representativeness. When a company announces surprisingly good earnings, conservatism means that investors react insufficiently, pushing the price up too little. Since the price is too low, subsequent returns will be higher on average, thereby generating both post-earnings announcement drift and momentum. After a series of good earnings announcements, though, representativeness causes people to overreact and push the price up too high. Since the price is now too high, subsequent returns are too low on average, thereby generating long-term reversals and a scaled-price ratio effect.

2. Overconfidence. Daniel, Hirshleifer and Subrahmanyam (1998, 2001) stress biases in the interpretation of private, rather than public information. They assume that investors are more likely to be overconfident about private information they have worked hard to generate than about public information. If the private information is positive, overconfidence means that investors will push prices up too far relative to fundamentals. Future public information will slowly pull prices back to their correct value, thus generating long-term reversals and a scaled-price effect. To get momentum and a post-earnings announcement effect, DHS assume so

called self-attribution bias: public news which confirms the investor's research strongly increases the confidence he has in that research; disconfirming public news, though, is given less attention, and the investor's confidence in the private information remains unchanged. This asymmetric response means that initial overconfidence is on average followed by even greater overconfidence, generating momentum.

3. Bounded rationality. Positive feedback trading plays a central role in the model of Hong and Stein (1999), where two boundedly rational groups of investors interact, meaning that investors are only able to process a subset of available information. "Newswatchers" make forecasts based only on private information, while "Momentum traders" condition only on the most recent price change. They assume that private information diffuses slowly through the population of newswatchers. By buying, momentum traders hope to profit from the continued diffusion of information. This behaviour preserves momentum, but also generates price reversals: since momentum traders cannot observe the extent of news diffusion, they keep buying even after price has reached fundamental value, generating an overreaction that is only later reversed.

4. Models with Institutional Frictions. The institutional friction that has attracted the most attention is short-sale constraints. They can make investors less willing to establish a short position than a long one. Several papers argue that when investors differ in their beliefs, the existence of short-sale constraints can generate deviations from fundamental value and in particular, explain why stocks with high price-earnings ratios earn lower average returns in the cross-section. There are at least two mechanisms through which differences of opinion and short-sale constraints can generate price-earnings ratios that are too high, and thereby explain why price-earnings ratios predict returns in the cross-section.

Miller (1977) notes that when investors hold different views about a stock, those with bullish opinions will, of course, take long positions. Bearish investors, on the other hand, want to short the stock, but being unable to do so, they sit out of the market. Stock prices therefore reflect only the opinions of the most optimistic investors, which, in turn, means that they are too high and that they will be followed by lower returns.

Scheinkman and Xiong (2001) argue that in a dynamic setting, a second, speculation-based mechanism arises. They show that when there are differences in beliefs, investors will be happy to buy a stock for more than its fundamental value in anticipation of being able to sell it later to other investors even more optimistic than themselves. Short-sale constraints are very important here, because in their absence, an investor can profit from another's greater optimism by simply shorting the stock. With short-sale constraints, the only way to do so is to buy the stock first, and then sell it on later.

Preference-based behavioural models.

Barberis and Huang (2001) show that application of loss aversion and narrow framing to individual stocks can generate the evidence on long-term reversals and on scaled-price ratios. The key idea is that when investors hold a number of different stocks, narrow framing may induce them to derive utility from gains and losses in the value of *individual* stocks. The investor is loss averse over individual stock fluctuations and the pain of a loss on a specific stock depends on that stock's past performance.

To see how this model generates a value premium, consider a stock, which has had poor returns several periods in a row. Precisely because the investor focuses on individual stock gains and losses, he finds this painful and becomes especially sensitive to the possibility of further losses on the stock. In effect, he perceives the stock as riskier, and discounts its future cash flows at a higher rate: this lowers its price-earnings ratio and leads to higher subsequent returns, generating a value premium.

## 2.3. Behavioural Trading Strategies

In this section we are illustrating two behavioural trading strategies: momentum and contrarian strategies, which are already being successfully used by some investors. The empirical evidence explaining momentum and reversal effects is given above as well as some behavioural applications to these phenomena. Below we summarize this information and explain strategies, which can be used to exploit these market inefficiencies.

### 2.3.1.Momentum Trading Strategies

Price momentum can be explained by the following behavioural factors:

1. Representativeness, which means that naïve investors extrapolate future earnings on the basis of the recent past. Expecting that stocks will continue to behave the way they did for, let’s say,

lest six months, investors may decide to take long positions on stocks having performed well, leading to price increases, and to take short positions on past loosers, leading to price decrease.

2. Overconfidence can also partially explain momentum, because many investors are more confident in their privately obtained information, then in information, which is publicly available. If public information contradicts private, most investors tend to underreact to this information, while if it supports private information; investor's overconfidence grows to even higher degree, coursing overreaction.
3. Private information diffuses among agents on the market gradually, coursing graduate price increase. Momentum traders may further provoke momentum by buying stocks in trend, but being unable to precisely evaluate the degree of information diffusion, may push prices higher then their fundamental value is, which will course the future reversal.
4. Short-sales constraints and different beliefs of investors can also explain momentum, because while bullish investors are buying stocks, bearish investors face difficulties in short selling them.

Momentum investing.

To implement momentum trading strategy, the first thing to do is to rank available stocks. To do so, it's necessary to define measures of price momentum. Empirical evidence has shown, that the best results from forming price momentum portfolios is obtained, when the period for ranking stocks lies somewhere between 6 to 12 months.

With price momentum, the bottom ranked stocks are those, that have realized the lowest return over the measurement period (referred to as "losers"), while the top ranked stocks are those that have realized the highest return ("winners").

The portfolio is formed basing on expectation that the winners will continue to outperform the loosers over the next several months.

### 2.3.2. Contrarian Trading Strategies

According to empirical evidence, price reversals take place after 2 or 3 years after portfolio formation. If a price reversal exists, it should be possible to implement a strategy, which allows capturing the advantages of a possible mispricing at a particular moment. Such a strategy is the so-called contrarian (or value) strategy.

<u>There are two possible explanations of outperformance of value strategies:</u>

1. First relies on the same belief as momentum effect - investors behave naively and base their expectations and forecasts in extrapolating information from past earnings and returns. Many investors tend to behave excessively optimistic towards stocks having well performed in the recent past and, at the same time, they are pessimistic on stocks having recently poorly performed. In doing so, investors overreact to the information flow and invest in these naive strategies. More attentive investors implement contrarian strategies, consisting in a bet against the naive investors. This suggests that value strategies yield positive returns because of the exploitation of sub-optimal behaviour of investors.
2. An alternative explanation for the outperformance of value strategies argues that investors rely excessively on analysts' long-term earnings forecasts, which in many cases reveal a too optimistic view. In the same way as a naive strategy based on the extrapolation of past earnings, investors observe the forecasts of financial analysts and agree to buy stocks which are predicted to grow, moving up their price and sell forecasted loser stocks moving down their price. Contrarian investors bet against naive investors and take positions, which are the opposite to those indicated by financial analysts. They would realize higher profits because they invest in undervalued stocks and short overvalued stocks.

Contrarian investing.

When choosing stocks for the strategy, good criteria are their market-to-book ratio and price-to-earnings ratio. A low M/B indicates, that the market value of a firm is low in comparison to its most recent book value. The reasons for a low M/B are represented by a poor performance of the stocks in the past and/or pessimistic forecasts on the future earnings of the firm. Thus, a high (low) M/B or P/E ratio is taken as indicative that the firm's stock is expensive (cheap). To form a value portfolio, contrarian investors are buying stocks whose prices are low and which are expected to underperform the market and selling the stocks whose prices are high.

This strategy is riskier then momentum strategy, but it can also provide higher returns. It was proven empirically, that if not one, but several criteria are used in ranking of value stocks, the performance of portfolio improves significantly.

### 2.3.3. Interplay Between Momentum And Contrarian Strategies

While evidence supports the success of contrarian and momentum strategies when practiced individually, there is the possibility that even better returns might be realized by combining them within a single investment strategy. With momentum we have a strategy that functions very well in trending markets, with contrarian, we have a strategy which performs very poorly when market valuations reach excesses towards the end of a strong bull market but which come into their own when prices revert back to more sustainable levels. The fact that added value from momentum is pro-cyclical, while that from value tend to be counter-cyclical raises the possibility of either combining them within a single portfolio or running them as separate streams within the one investment strategy.

Momentum and contrarian investing are very much part of the phenomenon with underreaction to individual pieces of information being an important aspect of trending markets while an overreaction to a series of similar announcements (e.g. good news) being an important contributor to the excesses in pricing which is what eventually gives rise to the conditions for contrarian investing to succeed.

An explanation provided by Hong and Stein (1999) provides insights as to benefits from an investment strategy that combines both value and momentum investing. These authors assume that the world consists of two types of investors: fundamental investors who act on news announcements and momentum investors who follow trends. In response to the initial piece of good news, the news followers drive up the price slightly and would continue to do so after the release of subsequent good news announcements. Thus a trend in pricing is created which increasingly attracts the trend followers into the stock, and so drives up the price even more. When the first piece of bad news arrives, the trend followers completely ignore it but the fundamental investors do put a break on the upward movement in price and will continue to sell the stock in reaction to subsequent bad news announcements. A negative trend is eventually created which again attracts the trend followers to sell and so further precipitates the fall in price to what is likely to now prove an unsustainable low level.

## 2.4. PERSPECTIVES IN BEHAVIOURAL FINANCE

Although, there are many recent papers on behavioural finance, much of the work here is narrow. Models typically capture something about investors' beliefs, or their preferences, or the limits of arbitrage, but not all three. As progress is made, more theories will emerge, which will be able to incorporate more than one strand.

For example, the empirical literature repeatedly finds that the asset pricing anomalies are more pronounced in small and mid-cap stocks than in the large cap sector. It seems likely that this finding reflects limits of arbitrage: the costs of trading smaller stocks are higher, and the low liquidity keeps many potential arbitrageurs uninterested. While this observation may be an obvious one, it has not, found its way into formal models. Interplay between limits of arbitrage and cognitive biases may become an important research area in the coming years.

Some of the institutional barriers, such as those regarding short selling, may also have behavioural explanations. Bringing institutions more directly into the behavioural model and applying the behavioural model to institutions will be hard but worth doing.

Most of the research so far has been in the field of asset pricing; much less has been done on corporate finance recently. One example of the kind of research that it might be possible to do in the realm of behavioural corporate finance is Jeremy Stein's (1996) article "Rational Capital Budgeting in an Irrational World." Stein ponders how companies should make investment decisions if asset prices are not set rationally. Many other papers, both theoretical and empirical, are waiting to be written in this important area.

Finally, more data on individual investors is necessary to better understand individual investors' behaviour. Similarly, tracking the behaviour of investors in 401(k)-type pension plans is of growing importance. For both cases, the data exist in the files of private firms, which are reluctant to share the information. For sharing to become a reality, confidentiality will have to be adequately protected - confidentiality of the source of the data and of the identities of the individual investors.

# 3. Statistical Arbitrage: Tool To Exploit Predictable Component Of Equity Returns

## 3.1. Hedge Funds And Their Strategies

### 3.1.1. The Hedge Fund Industry Overview

A hedge fund is a special type of investment vehicle, primary used by wealthy institutions and individuals, who pool their capital in order to implement high-risk skill-based investment strategies, financial instruments, investment styles, which are usually unavailable to other funds, i.e. mutual funds, which are limited to long positions. These strategies are mostly based on heavy leverage, short selling, and use of derivatives. A manager of a hedge fund who commits a part of his net worth (property, belongings) into the fund is compensated based on the percentage of a hedge fund's performance. The number of participants in a hedge fund is restricted by law to no more than 100 per fund. Consequently, most hedge funds have set very high minimum participation investment amounts, which starts from over $250 thousands.

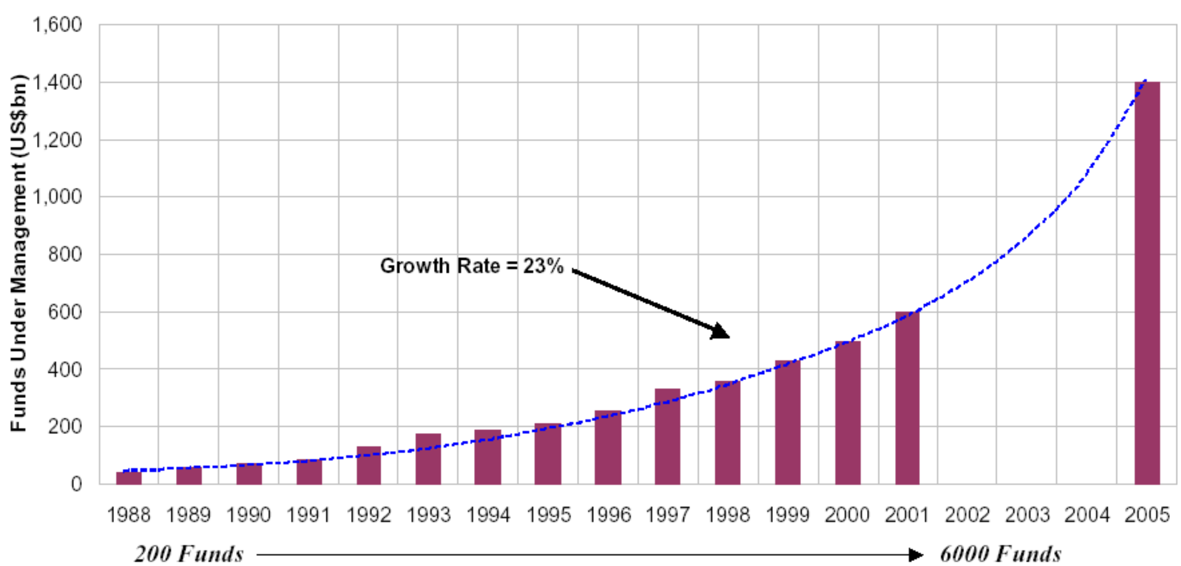
Hedge fund industry can be viewed as being flexible to make money in all market conditions (increasing and decreasing), preserving capital in falling markets (due to low correlation with market), not constrained with benchmarks, tracking errors and regulations that are able to prevent maximizing returns, and are talent- and experience-concentrated.

The idea to hedge against future price fluctuations belongs to the farmers in the United Stated who sold their crops and cattle against future delivery before harvesting them. Therefore, the farmers eliminated or reduced their market risk exposure by locking-in the price in advance. In the earlier 1950's, after gathering the materials about trends in investing and market forecasting, A.W. Jones came up with concept to use hedging techniques on equity markets. His idea was in order to reduce or eliminate the portfolio's risk borne by the long position one should short other stocks that have similar risk-return profile as long stocks. To increase the upside potential of that strategy he used leverage. Later, Jones decided to switch from general partnership to limited partnership, and began to charge all partners with 20% incentive fee, while leaving the part of his net worth in his fund sharing all risks. These changes became standards in the hedge fund industry.

The long/short strategy became very popular after the article about the Jones' fund was published in the Fortune Magazine in 1966. That article caused a sensation in the finance world; the Jones' fund outperformed "that year the best mutual fund by 44% and the best five-year performing mutual fund by 85%"[1]. Many investors struggling for high risk-premium decided to invest in hedge funds. However, in reality most of the hedge funds at that time did not really hedge their heavy long side portfolio's risk exposure supported by leverage with shortening other equity leaving them vulnerable to the equities price fluctuation. Such a risky position could not last long without any loss. According to Gary Spitz the number of the hedge funds decreased from 200 in 1968 to 85 in 1984.

And only since 1990's the industry became to grow very fast. Starting from around 230-odd funds in 1990 with $6.5 billion assets under their management, their number increased drastically. Today, according to Hedgeeco database, the number of hedge funds increased in more than 30 times to 7000 with estimated $400-500 billion in capital[2]. On the figure below one can see the evolution of the hedge fund industry.

Figure 3.1. Growth Of The Hedge Fund Industry[3]




Although the mutual fund industry is much bigger and the total volume of assets under their management exceeds that of hedge funds, the level of growth of hedge fund industry reflects the tendency of institutions and wealthy individuals toward alternative investments, because of their low correlation or even uncorrelation with traditional investments. Therefore, it allows them to diversify their investment portfolios and improve their risk-return profile. According to the statistics presented by Friendland, hedge funds significantly outperformed mutual funds (as representatives of traditional

[1] Gary Spitz, HedgeFund-Index.com
[2] D. Friedland, the chairman of the Magnum Fund
[3] Altmann R. 2002. Lecture Notes

investments) in falling equity markets. From 1990 S&P and average U.S. equity mutual fund had 15 and 14 negative quarterly returns respectively. Such a performance for almost 13 years leads them to have a total return of –108.12% and –111.8% respectively. Yet the average hedge fund experienced only with 9 quarterly negative returns, totalling a negative return of only –9.2%, proving its ability to perform well in falling equity markets.

Over the period from 1990 to mid-2002 HFRI Fund Weighted Composite had around 15% annualised return with bond-like annual volatility around 7.2%, while such equity indices as S&P Composite, FTSE 100, and MSCI World Index had much lower average annual return and much higher average annualised volatility. S&P Composite with around 9.2% had the highest return among them, and FTSE with around 14.2% had the lowest volatility.

Unlike mutual funds which have SEC regulation and disclosure requirements, hedge funds are much more flexible in their investment options. They can use short selling, leverage, derivative, and futures. Hedge fund industry attracts the best brains in the investment business because of the high remuneration award based on fund's performance.

There is no strict classification of the hedge funds within the industry based on the strategy the particular fund implements. This proves that these strategies are difficult to classify. Below we present Morgan Stanley's classification, however CSFB/Tremont and HFI classifications are used more frequently.

Table 3.1. The MSCI Hedge Fund Classification Standard[4]

| Specialist Credit | Directional Trading | Relative Value | Security Selection | Multi-Process Group |
|---|---|---|---|---|
| Distressed Securities | Discretionary trading<br>• Currencies<br>• Equity<br>• Diversified | Arbitrage<br>• Convertibles<br>• Fixed-income (MBS, ex MBS)<br>• Equity | No Bias<br>• Europe<br>• North America<br>• Diversified<br>• Japan | Event-driven |
| Long-Short Credit | Tactical Allocation | Merger Arbitrage | Short Bias | Multi-process |

[4] Morgan Stanley, Investable Hedge Fund Indices Methodology, June 2003

| | | | | |
|---|---|---|---|---|
| Private Placement | Systematic Trading<br>• Currencies<br>• Diversified | Statistical Arbitrage<br>• Europe<br>• North America | Long Bias<br>• Europe<br>• North America<br>• Diversified<br>• Japan<br>• Emerging Markets<br>• Global Markets Asia<br>• Asia ex Japan | |
| | | | Variable Bias<br>• Europe<br>• North America Diversified | |

Table 3.2. Hedge Fund's Strengths And Weaknesses

| Strengths | Weaknesses |
|---|---|
| Sustainable good performance | Lack of transparency in terms of strategies |
| High risk adjusted returns | Risk of failure due to high leverage |
| Motivated bright managers | Capacity constraints |
| Greater flexibility of investment instruments | Complex performance evaluation |
| Pro-active approach to investing | Large variations in individual performance |

### 3.1.2. HEDGE FUND STRATEGIES

Hedge funds implement different strategies that are grouped according to the common features-specific characteristics.

Table 3.3. Hedge Fund Investment Styles

| Long/Short Equity | Event Driven | Relative Value/ Market Neutral | Global Asset Allocation |
|---|---|---|---|
| Description:<br><br>Directional strategies involving equity oriented investing in both the long and short sides of the market | Description:<br><br>Strategies that can benefit from the occurrence of special situations | Description:<br><br>Strategies aiming to profit by capitalizing on the mispricing of related securities of financial instruments | Description:<br><br>Diverse mix of strategies/instruments that are generally momentum based over short holding periods |
| Sub-strategies:<br><br>• Value/Growth<br>• Sector<br>• Geographical<br>• Opportunistic<br>• Short Selling | Sub-strategies:<br><br>• Merger Arbitrage<br>• Distresses Securities<br>• Corporate Restructuring | Sub-strategies:<br><br>• Convertible Arbitrage<br>• Fixed income Arbitrage<br>• Statistical Arbitrage | Sub-strategies:<br><br>• Futures Trading<br>• Global Macro<br>• CTA |
| Features:<br><br>• Largest strategy<br>• Generally low leverage | Features:<br><br>• Low market exposure<br>• Probabilistic models | Features:<br><br>• Very low market exposure<br>• Arbitrage anomalies | Features:<br><br>• No correlation with MSI<br>• More volatile |

Long/short equity funds invest equally in long and short positions generally in the same sector of economy (for example, construction, aircraft, or hardware) or region, achieving market neutrality. They have the ability to shift from value to growth, from small-, mid-cap and to big-cap stocks, they can switch from a net long to a net short position. While implementing this strategy, hedge fund can use market index futures and options to reduce or eliminate systematic risks of its positions.

Event driven strategies aim to benefit from special situations or significant restructuring events such as spin-offs, mergers and acquisitions, bankruptcy reorganizations, capital restructuring and share buybacks.

Global Macro funds are the biggest in the industry. Changes in global economies, driven by changes in government policy which influences interest rates, in turn affecting currency, stock and prices, are the targets of these funds. They are usually highly volatile. The most famous are George Soros's Quantum Fund, Julian Robertson's Jaguar Fund, Leon Cooperman's Omega Overseas, Louis Bacon's Moore Global, and Mark Kingdon's Kingdon Fund.

Market neutral strategy inscribes many sub-strategies designed to benefit both in bull and in bear markets, which allows them to generate positive return when market goes up or down. They bet on spread relationships between financial assets or commodities.

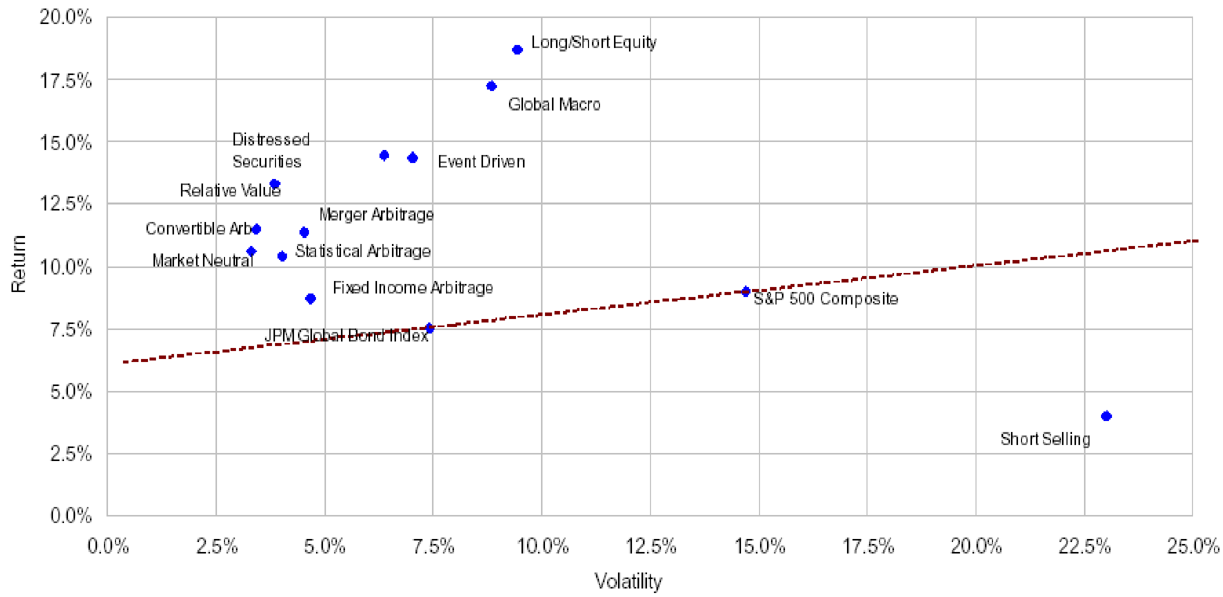
Every investment manager in the hedge fund implementing an active trading strategy wants to achieve the return, which is higher than that of passive buy-and-hold investments. The additional return is sometimes referred to as alpha[5] [Morgan Stanley, Quantitative strategies, 2000].

Table 3.4. Hedge Fund Risk And Return Characteristics (January 1990-July 2002)

| Strategy | Sub-strategy | Annualised Return | Volatility | Correlation | | |
|---|---|---|---|---|---|---|
| | | | | S&P 500 | MSCI | LBI |
| Long/Short Equity | Long/Short Equity | 18.72% | 9.44% | 0.66 | 0.62 | 0.14 |
| | Sector Specialist | 19.71% | 14.55% | 0.57 | 0.55 | 0.07 |
| | Short Selling | 4.02% | 23.01% | -069 | -0.67 | -0.10 |
| Event Driven | Merger Arbitrage | 11.39% | 4.54% | 0.46 | 0.41 | 0.10 |
| | Distressed Securities | 14.50% | 6.38% | 0.39 | 0.35 | 0.10 |
| Relative Value | Convertible Arbitrage | 11.51% | 3.41 | 0.35 | 0.33 | 0.10 |
| | Fixed Income Arbitrage | 8.75% | 4.68% | -0.06 | 0.00 | -0.07 |
| | *Statistical Arbitrage* | *10.36%* | *4.00%* | *0.53* | *0.43* | *0.34* |
| Global Asset Allocation | Global Macro | 17.24% | 8.84% | 0.42 | 0.42 | 0.36 |

[5] Morgan Stanley, Quantitative strategies, 2000

Figure 3.2. Out-performance Of Hedge Fund Strategies[6]




*Adding alpha in all market conditions (January 1990-July 2002)*

Hedge fund indices primarily evolved in response to increased number of hedge funds and strategies they implement and the desire to have an industry- and strategy-specific benchmarks, against which it could be possible to compare or analyse the performance of the certain strategy (hedge fund manager) or the fund as a portfolio of strategies. The problem with using benchmarks in this industry stems from dependency of the fund's performance upon individual skills of a manager, which no index could measure. The first hedge fund indices fail to capture strategy and sub-strategy-specific risk-return characteristics. As a response to the industry growth, main index providers began to separate the different hedge fund strategies and styles.

Major hedge fund industry index providers[7]:

1. Latest produces 14 indices based on the information provided by 2000 funds since 1993;
2. CSFB/Tremont calculates 11 indices quarterly using TASS database, which includes 2600 US and offshore hedge funds;
3. Evaluation Associates Capital Markets (EACM) calculates indices for five broad strategies and 13 underlying sub-strategies, using the data since 1990;
4. HedgeFund.net (Tuna) computes so called Tuna indices (33 indices) using the information

[6] Altmann R. 2002. Lecture Notes
[7] F.S. Lhabitant "Hedge Funds – Myths and Limits", 2002

from 1800 hedge funds since 1979.

5. Hedge Fund Research (HFR) produces monthly 33 indices using data from 1990, and since 2000 it calculates daily five indices: convertible bond arbitrage, equity hedge, event-driven, merger arbitrage, and distressed securities arbitrage;

6. Hennessey Group, LJH Global Investments, Van Hedge Fund Universe/Managed Account Reports LLC, ZCM/HFR Index Management, Zurich Capital Market;

7. Newcomers: Deutsche Bank Asset Management, Morgan Stanley Capital International (MSCI).

### 3.1.3. The Legal Environment of Hedge Funds

Normally, hedge funds are exempt from obeying the security acts, law and regulations that govern the issuance and trading of publicly traded securities passed by the Securities Exchange Commission (SEC).

Table 3.5. Legal Requirements And Exemptions For Hedge Funds

| Law | Target | Requirements | Reasons for exemption |
|---|---|---|---|
| Security Act of 1933 | Publicly Security Issuers | To register and to file reports with the SEC when publicly traded securities are issued | Hedge funds are considered to be a private placements |
| Security Exchange Act of 1934 | Security Brokerage Funds | To file reports and to maintain extensive records for broker dealers | Hedge funds are not security brokerage firms |
| Investment Company Act of 1940 | Mutual Funds | To register as an investment company, leverage, fees restriction, rules for investment diversification, obligatory information disclosure, profit distribution to shareholders each year | Hedge fund limit the number of investors giving preferences to institutions and wealthy individuals |

| Law | Target | Requirements | Reasons for exemption |
|---|---|---|---|
| Investment Advisers Act of 1940 | Investments Advisers | Restriction on fees structure, limits on investor's minimum wealth and investment portfolio value, compliance with SEC filing and registration requirements | Hedge funds usually do not give any advises to the general public |
| Commodity Exchange Act of 1974 | Individuals and Firms giving advise on futures trading | To register as commodity pool or CTA with the National Trading Commission, associated registration and information disclosure to CFTC is required | Not all hedge funds have the same organizational and operational structure as commodity pools or CTAs |

After the collapse of LTCM in September 1998, it became clear that the hedge fund industry could avoid regulation despite its highly sophistication and understanding of the risk involved. According to the Katz' systematic risk classification, LTCM collapse highlighted that hedge funds are subjects to two sources of risk: their default can cause losses on regulated entities, consequently these entities might be incapable to perform key economic functions, and the forced liquidation of collateral cannot compel third parties from involving in self-defence [Hedge Fund Regulation, Harvard Law School, 2002].

Policy makers suggested several ways how to reduce or eliminate (manage) exposure to hedge funds:

1. Regulation improvement of hedge fund's counterparties,
2. Transparency improvement about positions taken by hedge fund.

At the beginning some proposed to implement direct regulation of the hedge fund industry, but that proposition was not supported by the majority. Therefore, energy was concentrated on improvements in third-parties regulation. For example, the International Organization of Securities Commissions (IOSCO) proposed voluntary information disclosure by large hedge funds, and the Basel Committee preferred to have a Central Register of leveraged positions. There were other proposals based on incentives to record and disclose data, but they proposals did not concern offshore hedge funds. The debates highlighted the need for a coordinated approach by national regulators. It would be perfect if hedge funds could reveal the information about notional amount of their positions in each market. In such a case third parties has information only about the scale and location of positions without detailed specification of particular trades. Also disclosure of risk-return profile based on statistical analysis would be more effective and meaningful rather than listing specific security holdings and expecting that investors would perform risk analysis by their own. In addition to listed above, some solvency measures have to be established. Cash and capital relative to notional positions and the valuation of off-balance sheet assets and liabilities, using VaR methodology, proposed in the paper of Anthony H. Hanlon [Proposals for Reform of Hedge Fund Regulation, 2002], could measure solvency of a hedge fund. Hanlon predicts that it is probable that hedge fund managers will have to register with the SEC as investment advisors accreditation standards governing eligible investors may be raised.

### 3.1.4.HEDGE FUNDS IN EUROPE

Europe accounts for only about 15% of the world hedge fund market. The leading position in this industry belongs to the UK because of its strong asset management, local market research, and favourable regulatory environment. In addition, public investors in the UK have limited access to the hedge funds' products, and therefore, hedge funds on this market have low risk. Luxemburg and Switzerland occupy the next position. Recently, hedge funds have been established in France, Sweden, Ireland, and Italy. In most Distribution barriers, caused by different regulatory requirements, fiscal regimes, saving preferences, different cultural barriers are the features of the European hedge fund industry. In European countries national regulatory authority controls onshore hedge funds and their onshore marketers and managers, who give advises to hedge funds about investments strategies. Usually prime brokers and investors in European hedge funds are located onshore, because demand mostly comes from not only wealthy individuals and institutions but also from small investors, pensions and life insurers.

But in recent future the regulatory requirements will move toward American standards, which have less distribution barriers, so European hedge funds could compete with US funds. In September 2003 the European Parliament started a preliminary debate on hedge fund and derivative regulation, which probably will result in the first European directive on hedge fund regulation [Statman Consulting, Inc. Hedge Fund Regulation, 2003]. There is a lot of hope that this document will change the European hedge fund industry by introducing a fund passport, which means ones the fund has been established in one of the European countries it would have to comply with the directive to one regulator and would then be allowed to market the fund to customers throughout EU.

## 3.2. Statistical Arbitrage

### 3.2.1. Overview

Morgan Stanley, which was one of the biggest centres of statistical arbitrage in early 1980's, defines statistical arbitrage as model-based investment process, which aims to build long and short portfolios whose relative value is currently different from a theoretically or quantitatively predicted value. The constructed portfolios should represent industry, sector, market, and dollar neutrality [Hedge Fund Research, Inc.]. Statistical arbitrageurs are trying to profit from temporary deviations of equity prices from their fundamental value. They combine science (value theory, statistical decision theory, game theory, statistical pattern recognition techniques, time series techniques: autoregression, vector error correction, cointegration), skills and experience when implement statistical arbitrage. It is widely used by hedge funds, Wall Street companies, and even sophisticated independent investors. Many managers implement this strategy with a directional, typically long, bias.

D. Beunza and D. Stark define statistical arbitrage as an art of association. By association they mean the construction of equivalence (comparability) of properties across different assets.

The statistical arbitrageurs (equity market neutral managers) use the information they gather very efficiently. For example, if arbitrageur takes a long position in some stock anticipating that its price will increase but it reality it does not or it does not perform that well, he can use that stock for short selling which would lead to smaller risk of the total portfolio the arbitrageur holds. The positive return of this strategy comes from two different sources. The first one is pretty obvious, it happens when the price of the stock from the long side of the portfolio goes up. The second one comes from the short position, but the strategy benefit if the price of shorted equities goes down, which means that arbitrageur can buy the stocks he owe at a lower price.

Many hedge funds implement this strategy for the following reasons:

- Returns of the strategy are independent and uncorrelated with the market,
- Volatility is pretty low,
- The strategy generates relatively high and constant return regardless of the economic downturns,

- The strategy complements other strategies used by hedge fund increasing portfolio diversification.

A major drawback of the strategy is that it becomes costly due to short selling and transaction costs. Another concern is the limited availability of stocks for short sale and strict rules, which prohibit short selling if the stock does not experience the previous up-tick. Since arbitrageur is looking for the highly liquid stocks to be short, it may happen that there are not enough stocks on the market. Such a problem is called capacity issue within the portfolio.

Statistical arbitrage is a more broad term than pure arbitrage, and unlike pure arbitrage that is riskless, statistical arbitrage bears the "risk to have negative payoff provided that the average payoff in each final state is nonnegative"[8].

S. Hogan, R. Jarrow, and M. Warachka extend the definition of statistical arbitrage. They emphasize that if the strategy is self-financing, zero-cost and generates cumulative discount profit that satisfies four conditions listed below, then such a strategy is statistical arbitrage.

1. At $t_0$ discounted profit is zero,
2. When t goes to infinity, expected discounted profit is strictly positive, which means that strategy at least should generate return equal to risk-free rate.
3. When t goes to infinity, the probability of having negative expected discounted profit equals zero, meaning that in the limit statistical arbitrage strategy converges to pure arbitrage.
4. When t goes to infinity, a time averaged variance converges to zero when there is positive probability of a loss at every finite point in time, which could be achieved through portfolio rebalancing or controlling the value of long and short positions to avoid excessive net exposure either long or short.

The fourth condition is crucial due to two reasons. Firstly, it distinguishes statistical arbitrage and pure arbitrage that satisfies the condition when the probability of loss at some point in time equals zero. Secondly, in the Black-Scholes economy statistical arbitrage defined only under 1-3 conditions is equivalent to buy and hold strategy.

They also tell the difference between statistical arbitrage and Ross' limiting arbitrage opportunity used in his APT model. "The difference between the two concepts is that statistical arbitrage is a

[8] Oleg Bondarenko, Statistical Arbitrage and Security Prices, The Review of Financial Studies, Fall 2003, Vol.16, No.3, p. 875

limiting condition across time, while Ross' APT is a cross-sectional limit at a point in time". In the financial theory we distinguish between weak and strong form of market efficiency based on what kind of information is reflected in stock prices. In a weak form of market efficiency stock prices incorporate only publicly available information, while in a strong form, stock prices reflect both publicly and privately available information. Therefore, if arbitrageurs in their models use public and private information they implement strong-form statistical arbitrage, otherwise they carry out weak from.

### 3.2.2. Statistical Arbitrage Trading Strategies

The variety of statistical arbitrage strategies is enormous and it's impossible to give the whole overview of them in this paper. However, we would like to mention some groups of trading strategies, used by hedge funds to implement statistical arbitrage.

1. Pair/Basket Trading

Pair trading, also known as spread trading, is a statistical arbitrage strategy that allows the trader to capture anomalies, relative strength or even fundamental differences on two stocks or baskets of stocks while maintaining a market neutral position.

The strategy may be implemented through matching a long position with a short position in two stocks in the exact same sector. This creates a hedge against the sector and the overall market that the two stocks are in. The hedge created is essentially a bet that you are placing on the two stocks; the stock you are long in versus the stock that you are short in. It's the ultimate strategy for stock pickers, because stock picking is all that counts. What the actual market does won't matter much. If the market or the sector moves in one direction or the other, the gain on the long stock is offset by a loss on the short. The profit comes from the changes in spread between the two. Therefore, the bet is being placed not on which direction the stock market will move, but on company-specific or sector-specific correlations.

2. Multi-factor models

To this group of statistical arbitrage models belong strategies, which are based on correlations of stock returns with several factors chosen. The best example of such model is Arbitrage Pricing

Theory. The strategy consists in defining factors, which influence stock returns, running multiple regressions on those factors and picking the stocks for portfolio on the basis of their respective correlations.

3. Mean-reverting strategies

This type of strategies is based on the assumption, that the stock prices are mean-reverting. So, if the stock price deviates from its supposed average value, it's expected to move in the future in the opposite direction. According to the strategy, the outperforming stock (expected to decrease in the future) should be sold short while the underperforming stock (expected to increase) should be bought. One of the examples of this type of strategies is contrarian trading.

4. Cointegration

The applicability of the cointegration technique to asset allocation was pioneered by Lucas (1997) and Alexander (1999). Its key characteristics, i.e. mean reverting tracking error, enhanced weights stability and better use of the information comprised in the stock prices, allow a flexible design of various funded and self-financing trading strategies, from index and enhanced index tracking, to long-short market neutral and alpha transfer techniques. A number of trading strategies can be constructed based on cointegration relationships:

1. Index tracking

The first cointegration-based trading strategy investigated is a classical index tracking aiming to replicate a benchmark in terms of returns and volatility. An index tracking process entails two, equally important stages: first, selecting the stocks to be included in the tracking portfolio and second, determining the portfolio holdings in each stock based on a cointegration optimization technique.

2. Enhanced index tracking and statistical arbitrage

Having constructed the simple tracking strategy, a natural extension for exploiting the tracking potential of the cointegrated portfolios would be to replicate artificial indexes, 'plus' or 'minus', constructed as to linearly over-perform or under-perform the market index by a given amount per annum. Then, self-financing long-short strategies can be set up by being short on a portfolio tracking the 'minus' benchmark, and long on a portfolio tracking the 'plus' benchmark. This type of statistical arbitrage strategy should generate returns according to the 'plus'/'minus' spread (i.e. double alpha) with fairly low volatility and no significant correlation with the market returns.

The cointegration relationship between the market index and its component stocks has a solid rationale, but this is not necessarily the case for portfolios tracking artificial benchmarks, which may be chosen to over-perform the market index by 50%, for example. The difficulty in finding an appropriate cointegration relationship leads to an increased instability of the stock weights, higher transaction costs and higher volatility of returns. To avoid this, it is essential to ensure that all the portfolios tracking 'plus' or 'minus' benchmarks pass the cointegration test.

Long and short portfolios formed under market neutral condition can be considered as a synthetic asset, which any price deviation from zero represents mispricing, and thus, possibility of statistical arbitrage strategy to be profitable. Profitability of this strategy arises because stock price deviates from random walk as supported by the empirical evidence. This deviation represents mispricing in statistical arbitrage sense and points out that there is predictable component in the price-dynamics [A.N. Burgess, 1999]. In his paper “Statistical Arbitrage Models of FTSE 100" Burgess proposes three-stage methodology to exploit statistical arbitrage:

1. The portfolio of long and short positions is constructed and is tested for existence of predictable component in the price-dynamics,
2. The error-correction mechanism is modelled between relative prices,
3. The statistical arbitrage strategy is used to benefit from having predictable component in equities returns.

Burgess improved the standard cointergation methodology in the following ways:

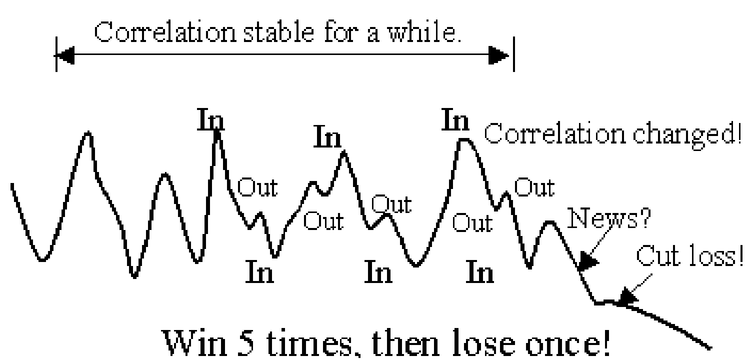
1. Cointegration test for stationarity he replaced with variance test for predictability, which is more appropriate for identifying statistical arbitrage opportunities;
2. Standard regression or principal component analysis he replaced with the stepwise regression, which is more reliable to deal with the highly dimensional samples.

His statistical arbitrage model uses stock “mispricings” (cointegration residuals) and lagged returns to predict relative returns on a one-day basis. He found that his generalized cointegration approach works very well with statistical arbitrage. The model generates profit in 85% cases during the period between June 1996 and May 1993 without transaction costs, and in 67% cases after introducing transaction costs at a level of 50 bp (0.5%). In the first case the Sharpe ratio was 15.7, and in the second case it lowered twice.

As one of the examples of practical application of mathematical models in statistical arbitrage trading we would like to present market neutral relative value trading strategy from the Tradetreck company. This trading strategy is based on correlation analysis, pattern recognition, and stochastic control theory. The strategy generates on average an annualised return around 60%+/-17%. The Marker-Neutral Pair Trade Model is a web-based model of the original trading system that is called Smart Trader 60[9]. This system is better at job than conventional statistical arbitrage strategy since it defeats a couple of drawbacks that are statistical arbitrage-specific. It can reduce or eliminate confusing unexpected correlation, and dynamically recognize factors that influence predictability of drift using stochastic price signals. These are oscillation and mean-reversion. Such signal are generated by trading positions entered in a group of similar stocks on a buy lows and sell highs basis, eliminating the random component in stock price dynamic. Therefore, to profit one should follow optimal entry and exit strategies.

Classical statistical arbitrage consists in constructing non-trending price signals based on correlation analysis through first identifying securities that are mispriced against the internal model's benchmark (theoretically or quantitatively predicted), and then buying lows and sell highs with cutting losses if trades lose more than set targets. Its graphical representation is shown below.

Figure 3.3. Non-Trending Price Signals[10]




[9] The material is used from the official web-site of Tradetreck company
[10] www.tradetreck.com, 2001

# 4. Behavioural Statistical Arbitrage Strategy

## 4.1. Data description

We apply our models to the historical prices of the stocks constituting the German, French, and Swiss stock markets indices.

From the international database DataStream we obtained the mid-week closing dividend and splits adjusted price data covering the period from 02.01.1985 to 09.07.2003 for the Swiss and French markets, and from 03.07.1991 to 09.07.2003 for the German market. Middle week prices are taken to ameliorate issues related to the beginning and end of the week noise. These lengths of the sample periods are determined by data availability. The following table contains general description of the indices we work with.

Table 4.1. Data Description

| Country | Index name | Symbol | Current number of stocks | Sample period |
|---|---|---|---|---|
| Germany | DAX | GDAXI | 30 | 03.07.1991 - 09.07.2003 |
| France | CAC 40 | FCHI | 40 | 02.01.1985 - 09.07.2003 |
| Switzerland | SMI | SSMI | 27 | 02.01.1985 - 09.07.2003 |

The sample periods had to be modified before implementing the models. The issue is that new stocks are periodically included into the market indices throughout the sample period. As a result, some constituents have relatively short histories. We would like to construct a sufficiently long sample of stocks with full price history. This way we can assure that if we end up with persistent statistical arbitrage profits, these come from the model rather than from exploiting varying investment opportunities. As a result, we are left with 19 stocks from SMI index covering the period from 12.10.1988 to 09.07.2003, 29 stocks from CAC 40 index covering the period from 19.04.1989 to 09.07.2003 and 18 stocks from DAX index covering the period from 10.07.1991 to 09.07.2003. The performance of our models on each market is then compared to the weighted averages of respective stocks since the original indices can no longer be our benchmarks (adjusted market index).

## 4.2. METHODOLOGY OF THE STRATEGY

### 4.2.1. OVERVIEW OF PREVIOUSLY IMPLEMENTED MODELS

Larson, Larson and Arberg [2002] are testing a market-neutral statistical arbitrage model using the most liquid stocks from Swedish market over the period from 30.06.1995 to 06.11.2001. First, on "signal generation" phase, they use momentum techniques to create the list of stocks that exhibit the strongest momentum. All stocks are ranked on the basis of the following criteria: cumulative return during prior 6-month period (with an extra weight put for the last week), book-to-market ratio, magnitude of price change during increase in trade volume, one year ahead expectations of cash flow changes, and market capitalization (small/large caps). Then, these rankings are used to construct equally weighted long and short portfolios (each including 10 stocks).

Next, they move to the "risk control" phase. Four different categories of risk control are singled out. First category considers the portfolio volatility and portfolio correlation with other assets. Statistical arbitrage is generally considered to be a market neutral strategy, with low portfolio's volatility and low covariance between long and short positions; however, sometimes the low covariance condition is relaxed to bet on directional movements in long and short positions. Thus, total portfolio's beta is kept around zero, eliminating the risks that are correlated with the market. Even if this is achieved it is important to avoid having negatively correlated stocks in long and short portfolios. Following momentum effect phenomena, inclusion of growth versus value stocks and large caps versus small caps solves this problem. The growth stocks overweight value stocks in long and short portfolios and will therefore sustain portfolio stability. Portfolio systematic risk exposure is minimized with inclusion in the short portfolio large caps and in the long portfolio small caps. Ergo, the only problem that is left to be solved is the covariance problem within the portfolio. Larson, Larson and Arberg [2002] proposed the following solution: 4 best candidates for inclusion in the portfolio are tested by calculating the sum of the covariance matrix, one at a time, and the one that has the lowest sum is included. This way it is possible to find the stocks with strong momentum effect and favourable volatility.

The other three categories of risk control are the stop-loss rule, a low cut-off price, and an indicator of extreme valuations. The transaction costs for the long and short position are set at 0.3% per transaction. Rebalancing of the long and the short positions is required each time when either long or short positions exceeds the other by more than 25% (to keep portfolio neutrality). The composition of the portfolio is totally changed at the end of every 4-month holding period.

Table 4.2. Results Of The Trading Strategy:

| | |
|---|---|
| Annualised return | 21.8% |
| Daily standard deviation | 1.46% |
| Annualised standard deviation | 25.6% |
| Beta | -0.011 |
| Modified Sharpe ratio* | 0.85 |

This trading strategy was also tested on German, French and UK markets with the following settings: no transaction costs, no risk control, ranking is based exclusively on the basis of 6 month price change (without weighing the last week), rebalancing is done every fourth month. The results are shown in the table below.

Table 4.3. Results of the model on European markets

| | Germany | France | UK |
|---|---|---|---|
| Annualised return | 8.25% | 7.42% | 10.9% |
| Daily standard deviation | 1.22% | 2.24% | 1.32% |
| Annualised standard deviation | 22.5% | 42.7% | 27.7% |
| Modified Sharpe ratio | 0.37 | 0.17 | 0.39 |

### 4.2.2. Our Approach To Implementation Of The Strategy

Our approach is similar to the one used by Larson, Larson and Arberg [2002] on German, French and UK markets. We are working with a slightly different time frame and Swiss rather than UK index. Also, since we are concentrating on indices with the small number of stocks, we operate with smaller portfolios.

* Calculated as return per unit of volatility

The main contribution of our approach is that we consider behavioural statistical arbitrage models as constrained optimisation problems that would provide some desirable risk-return profile. The constraints include zero beta of the portfolio with respect to market and zero cost of the strategy. We are considering two different measures of risk: portfolio's variance, and the covariance between long and short positions.

Our methodology is based on the presumption that history is a good predictor of the future. Therefore, when portfolio's variance is taken to be a measure of risk, our goal is to construct a portfolio that has the lowest feasible historical volatility at the beginning of the holding period. On the other hand, when covariance between long and short positions is taken to be a measure of risk, we are looking for a portfolio that would have the lowest correlation between the positions within the total portfolio. By this we would eliminate price co-movements between the taken opposite positions. In both approaches we concentrate on minimization of risk rather than explicitly look for optimal risk-return combination. This methodology is justified by the fact that our portfolio constituents are already the stocks with the strongest momentum, so portfolio's return is expected to be high in any case. Besides, this allows us to avoid making assumptions about investor's utility function, which is inevitable in mean-variance optimisation.

We expect these approaches to give us the desired level of strategy profitability with some moderate level of risk. In addition, we hope that they will reach the performance of the statistical arbitrage index provided by Hedge Fund Research with its average annual risk-return profile presented in table 3.4.

Our algorithms consist of two stages. The first stage is similar to the signal generating stage of Larson, Larson and Arberg [2002]. At the beginning of each holding period we rank the stocks according to their past performances (the informative prior observation periods are taken to be 6 and 12 months). We rely on the momentum theory in that we expect stocks with relatively high (low) performance during prior 6 to 12 months to maintain the same lead (lag) over the next 4 to 12 months. The performance is measured on the basis of cumulative return, which was proved to be the most important variable in seeking the momentum effect. In each market, we pick five winners (stocks with the highest realized return over the measurement period) and five losers (stocks with the lowest realized returns) to be constituents of the long and short parts of the arbitrage portfolio respectively. This approach guarantees us to have the stocks in the portfolio with the strongest momentum effect. However, we understand that since the cross-sectional dimension of our samples is small, the resulting portfolios will not be well diversified.

Next, we proceed with optimisation under investment restrictions to construct zero-cost portfolio such that it has the lowest variance, or the lowest covariance between long and short positions from the pull of possible weight combinations (optimisation stage). To avoid overexposure and underinvestment to any of the equities included in the portfolio, we impose an extra requirement that each stock's weight should be within the lower bound of 10% and the upper bound of 60%. We assume that no rebalancing or any other adjustment to the portfolio is done during the holding period. We then measure the performance (return) of this portfolio assuming holding periods of 1, 4, and 6 months. These steps are repeated throughout the sample length.

Ideally, when measuring performance of our strategy, we should take into account various transaction costs, as well as costs related to establishing and managing margin accounts. But to simplify modelling, we relaxed all the costs in our empirical research.

## 4.3. Portfolio Simulation

### 4.3.1. Portfolio Variance Minimization Under Investment Constraints

The first approach that we consider is formalized as a variance minimization problem that reads as:

$$\max_{W} \; V = -w'\Sigma w,$$

$$\text{subject to } \sum_{i=1}^{N+M} w_i = 0,$$

$$\sum_{i=1}^{N} w^L{}_i = 1, \; \sum_{j=1}^{M} w^S{}_j = -1, \; 0.1 < w_i^L < 0.6, \; -0.6 < w_i^S < -0.1,$$

where $N$ and $M$ is the number of stocks in the long and short portfolio correspondingly, $\boldsymbol{\Sigma}$ is a variance-covariance matrix of the portfolio, $w^L$ indicates long portfolio allocation, $w^S$ indicates short portfolio allocation, and $w$ represents total portfolio allocation. In our case, $N=M=5$. Below, we will loosely call the goal function as "utility function". In this model we do not explicitly impose a market neutrality condition.

We consider long and short position as separate sub-portfolios, and implement the following procedure.

1. We start with an equally weighted allocation that satisfies our self-financing constraint.

2. Given that allocation, we compute the marginal utilities ($MU = - 2*\Sigma w$) from changing holding each of the equities separately for the long and short positions.

3. In the long and short parts we find the equities (candidates) with the lowest and the highest marginal utilities that are not on lower and upper bound correspondingly.

4. For each sub-portfolio, the increase in the weight of candidate stock with the highest marginal utility is equal to the decrease in the weight of candidate stock with the lowest marginal utility. The increases in utility from changing the holdings of the long and the short sub-portfolios are compared, and the one that brings the bigger increase is accepted.

5. We consider the optimal portfolio to be found when the difference between the highest MU and the lowest MU in each sub-portfolio is less than 0.0001. Otherwise we continue the procedure starting from step 2.

We determined the optimal amount by which the equity's weight with the highest MU should be increased and the weight of the equity with the worst MU should be decreased. The optimal change in the portfolio weights are found with respect to each position. New portfolio weights equal $w+c*s$, where $c$ is the optimal change either in long or short portfolios, and $s$ is a vector which has 1 for the equity which holdings are to be increased, -1 for the equity which holding are to be decreased, and 0 otherwise.

Therefore, we take first order conditions of the difference between new and old portfolio allocation $DV$ with respect to $c^L$ and $c^S$. For the long portfolio we have

$$\Delta V^L = -[w^L + c^L s^L; w^S]' \Sigma [w^L + c^L s^L; w^S] - (-w'\Sigma w),$$

and for the short portfolio we have

$$\Delta V^S = -[w^L; (w^S + c^S s^S)]' \Sigma [w^L; (w^S + c^S s^S)] - (-w'\Sigma w).$$

While calculating a new allocation, we have to make sure that investments stay within the boundaries. The violation of boundaries happens if the value for $c^L$ or $c^S$ exceeds $w_{U_J} - w_j$ or $w_i - w_{L_I}$. Thus, $c^L$ and $c^S$ equal:

$$c^L = \min\left( \frac{A_1 + B_1 A_2}{1 - B_1 B_2}, w_{U_J} - w_j, w_i - w^L{}_i \right),$$

$$c^S = \min\left( \frac{A_2 + B_2 A_1}{1 - B_1 B2}, w_{U_J} - w_j, w_i - w^S{}_j \right),$$

where

$A_1 = -\frac{s_L' \Sigma_1 w_L + w_L' \Sigma_2 s_L}{s_L' \Sigma_1 s_L}$, $B_1 = -\frac{s_L' \Sigma_2' s_S}{s_L' \Sigma_1 s_L}$, $A_2 = -\frac{s_S' \Sigma_2 w_L + w_S' \Sigma_3 s_S}{s_S' \Sigma_3 s_S}$, $B_2 = -\frac{s_S' \Sigma_2 s_L}{s_S' \Sigma_3 s_S}$, where $\Sigma_1$ is 5 by 5 matrix taken from the upper-left corner of the variance-covariance matrix, $\Sigma_2$ is 5 by 5 matrix taken from the lower-right corner of that matrix, and $\Sigma_3$ is 5 by 5 matrix taken from lower-left or upper-right corner of that matrix.

As mentioned above, this approach does not explicitly take into account the requirement of portfolio's market neutrality inherent in statistical arbitrage approach. To understand how closely the above model satisfies this requirement, we compute realized beta of our strategy with respect to weighted-average index of all available stocks.

To simplify beta estimation procedure, we used the standard regression estimator of the market return coefficient from[11]

$$\tilde{r}_j = a + b_j \tilde{r}_M + e_j .$$

The standard regression estimator will be of the form

$$\hat{b}_j = \frac{\hat{\sigma}_{jM}}{\hat{\sigma}_M^2} .$$

We wrote the code in the Matlab to implement the above procedure. We tested our model with different signal generating periods lasted from 6 months to one year. We also estimated how the model's profitability changes if the holding period changes. The motivation behind it was that the momentum persistence exists from 4 months to 1 year after a portfolio construction, and investors desire to react to changes in equity's return as soon as possible. Therefore, we used as a holding period one month, four months, and six months.

We performed six different strategies on Swiss, French, and German markets under portfolio

[11] Danthine,Donalson, Intermediate Financial Theory, 2002

variance minimization model. These models are classified according to the performance measurement periods: six-months and one year. For each of these groups we implemented above-mentioned holding periods.

The results of our first model tested on the Swiss market are displayed on figure 4.1., and the annualised statistics are shown on the table 4.4.

These results show that our model is profitable for all strategies and has the best performance measured with the Sharpe ratio under six-months estimation period and four months holding period afterwards. This result is natural according to the researches performed by Jegadeesh and Titman [1993]. Moreover, this strategy (six months measurement period and four months holding period) has the lowest annualised volatility equal to 18.5% whereas for all other strategies volatility stays within the range of 22-24%, and the lowest realized beta (-0.05). Therefore, this strategy fully agrees with the statistical arbitrage definition.

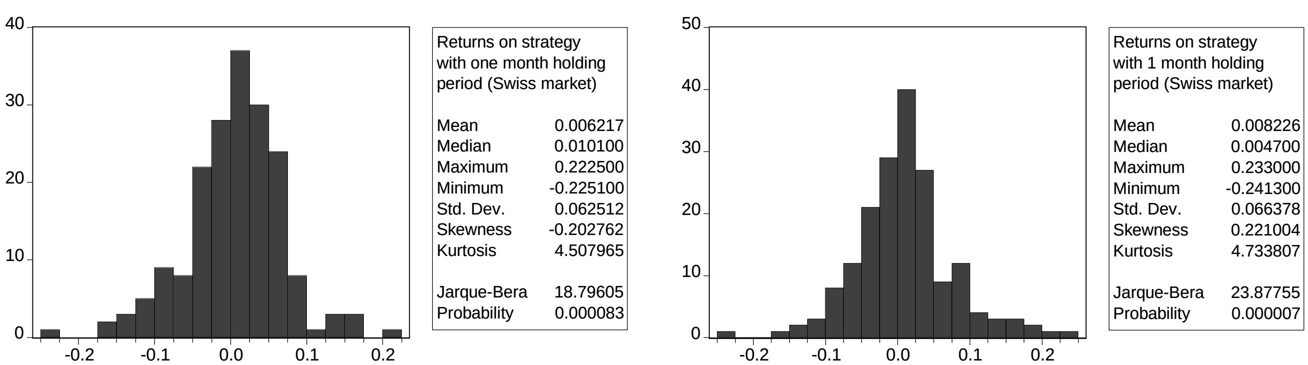
Under one-year measurement period the best performance has the strategy with one month holding period. This is not quite surprising since market preserves momentum effect on short-run (from 6 months to one year) and after 1 year the trend can change. Therefore, one month holding period is the best response to change in trend.

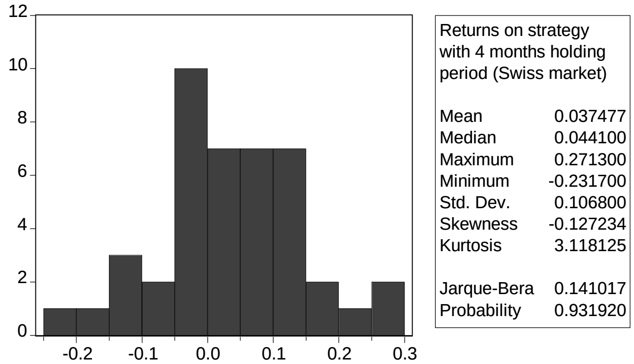
Measurement period extension is advisable for one-month holding period strategy, since the strategy's return increased more than it's volatility leading to higher Sharpe ratio.

Figure 4.1. Distribution Of Returns On Different Strategies On Swiss Market (Variance Minimisation)

6 months measurement period

1 year measurement period





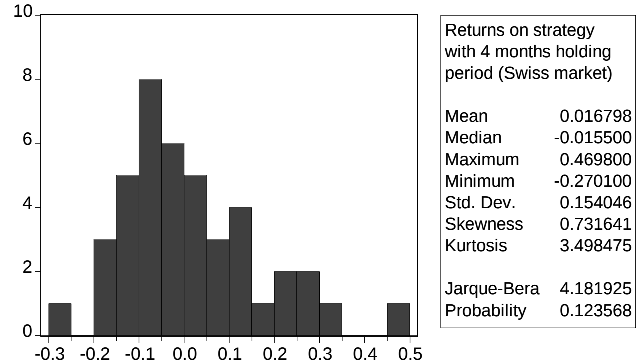




Table 4.4. Performance Of The Variance Minimization Model On Swiss Market

| Characteristics | 6-month measurement period | | | 1-year measurement period | | |
|---|---|---|---|---|---|---|
| | 1 month | 4 months | 6 months | 1 month | 4 months | 6 months |
| Annualised return | 7.93% | 11.24% | 8.94% | 10.49% | 5.04% | 8.05% |
| Annualised standard deviation | 22.32% | 18.5% | 22.75% | 23.7% | 26.68% | 21.95% |
| Beta | -0.26 | -0.05 | -0.09 | -0.37 | -0.21 | -0.24 |
| Modified Sharpe ratio | 0.36 | 0.61 | 0.393 | 0.4425 | 0.1889 | 0.3667 |
| Skewness | -0.2028 | -0.1272 | 0.1324 | 0.221 | 0.154 | 0.617 |
| Kurtosis | 4.508 | 3.118 | 2.229 | 4.7338 | 3.498 | 3.6255 |

The performance of Swiss market adjusted index over the periods the above strategies were implemented is shown on the figures 5 – 8, and the annualised characteristics are displayed in the table 3.4. We introduced the performance of the Swiss adjusted index and other market indices over the time periods the model’s strategies are implemented to make adequate comparison analysis.

Figure 4.2. Distribution Of Swiss Adjusted Market Index Returns Over Different Periods

6 months measurement period 1 year measurement period

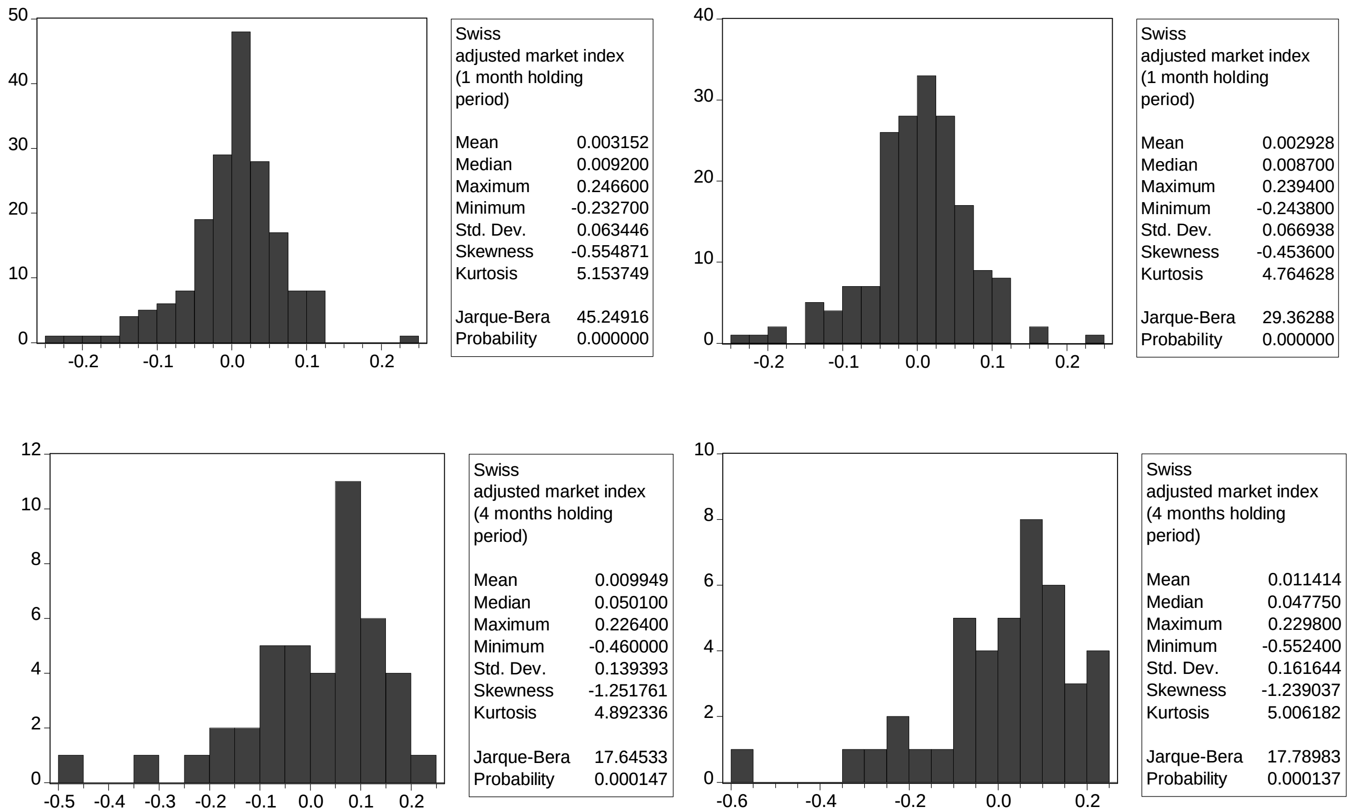



Table 4.5. Performance Of The Swiss Adjusted Market Index

| Characteristics | 6-month measurement period | | | 1-year measurement period | | |
|---|---|---|---|---|---|---|
| | 1 month | 4 months | 6 months | 1 month | 4 months | 6 months |
| Annualised return | 4.02% | 2.99% | 2.15% | 3.73% | 3.42% | 1.7% |
| Annualised standard deviation | 22.65% | 24.14% | 27.01% | 23.9% | 27.998% | 27.45% |
| Modified Sharpe ratio | 0.1774 | 0.1236 | 0.08 | 0.1562 | 0.1223 | 0.06 |
| Skewness | -0.55 | -1.25 | -0.71 | -0.45 | -1.24 | -0.669 |
| Kurtosis | 5.15 | 4.89 | 3.17 | 4.76 | 5.0 | 3.05 |

It is easy to notice that our model outperforms the market index. It has lower volatility and higher return.

The same strategies are implemented on French and German markets. The results of the model performance on these markets are displayed below.

Figure 4.3. Distribution Of Returns On Different Strategies On French Market (Variance Minimisation)

6 months measurement period | 1 year measurement period

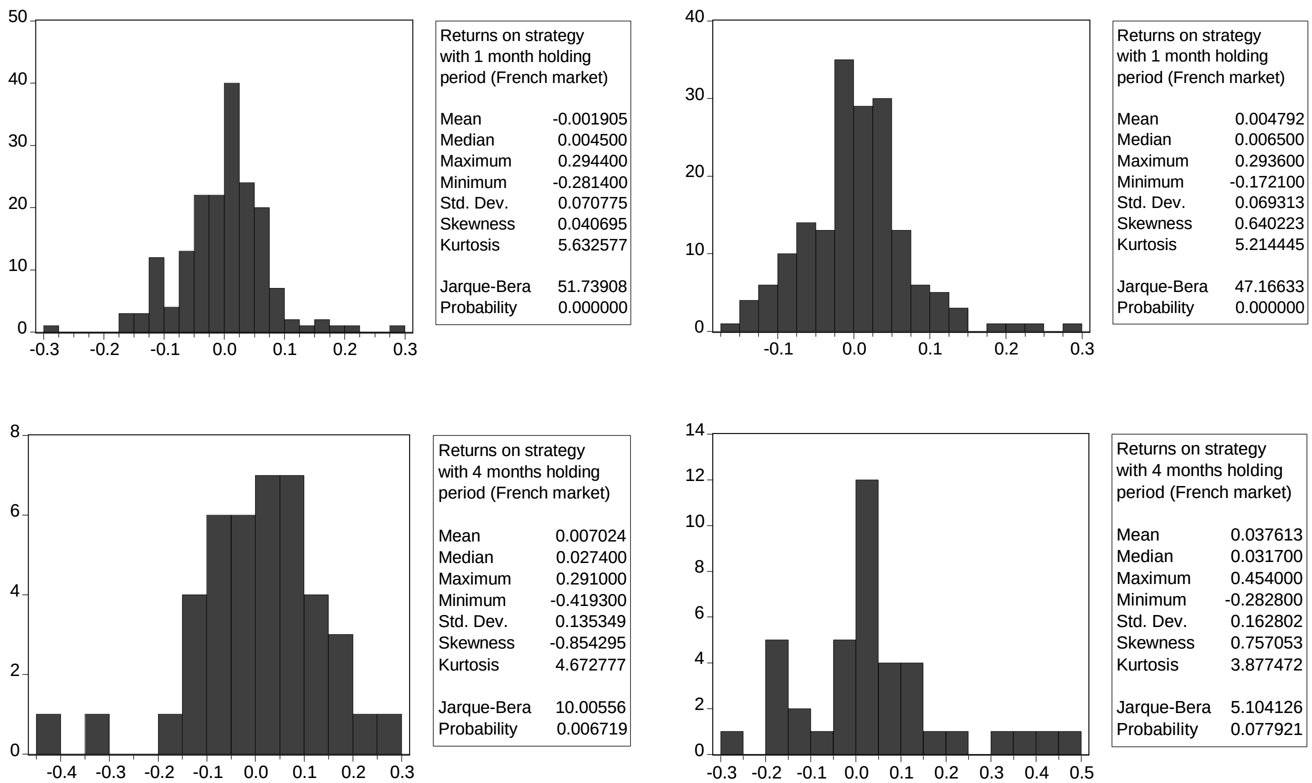



Table 4.6. Performance Of The Variance Minimization Model On French Market

| Characteristics | 6-month measurement period | | | 1-year measurement period | | |
|---|---|---|---|---|---|---|
| | 1 month | 4 months | 6 months | 1 month | 4 months | 6 months |
| Annualised return | -2.43% | 2.11% | 2.96% | 6.11% | 11.29% | 8.99% |
| Annualised standard deviation | 25.3% | 23.44% | 29.37% | 24.75% | 28.2% | 31.09% |
| Beta | -0.44 | 0.02 | -0.27 | -0.3 | -0.238 | -0.55 |

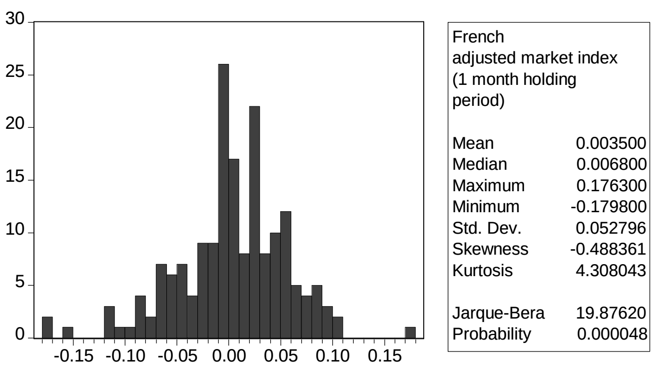
| | | | | | | |
|---|---|---|---|---|---|---|
| Modified Sharpe ratio | -0.10 | 0.09 | 0.10 | 0.25 | 0.4 | 0.29 |
| Skewness | 0.041 | -0.854 | -0.515 | 0.64 | 0.757 | 0.01 |
| Kurtosis | 5.633 | 4.673 | 2.573 | 5.21 | 3.877 | 3.44 |

On the French market, the model in five out of six strategies is profitable, and the strategy with one-year measurement period and four months holding period has the best performance. It has the Sharpe ratio of 0.4 and the second best strategy, with one-year measurement period and six months holding period, has it equal to 0.29.

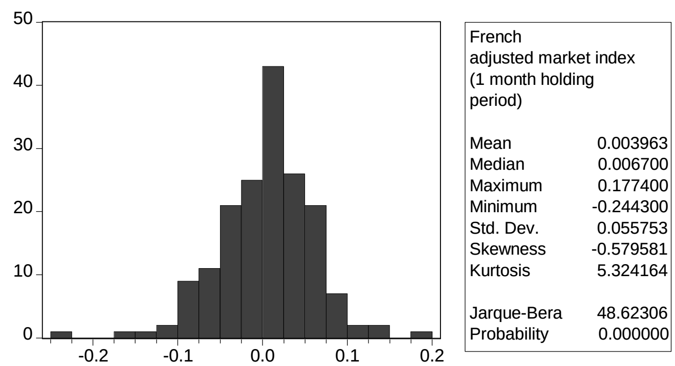
The strategies with one-year measurement period outperform the corresponding strategies with six-month measurement period. Negative sign of the first strategy (six-month measurement period and one-month holding period) suggests that contrarian (value) approach instead of momentum should be used. This means that stocks, which outperformed the market over the last six months, should be short, and the stocks, which underperformed the market, should be long. This way we could achieve positive return on our model under that strategy.

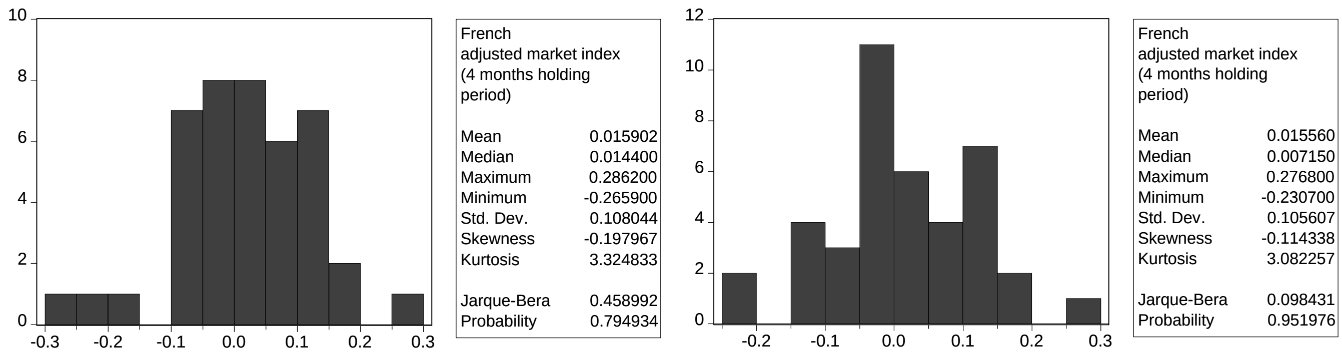
Figure 4.4. Distribution Of French Adjusted Market Index Returns Over Different Periods

6 months measurement period




1 year measurement period





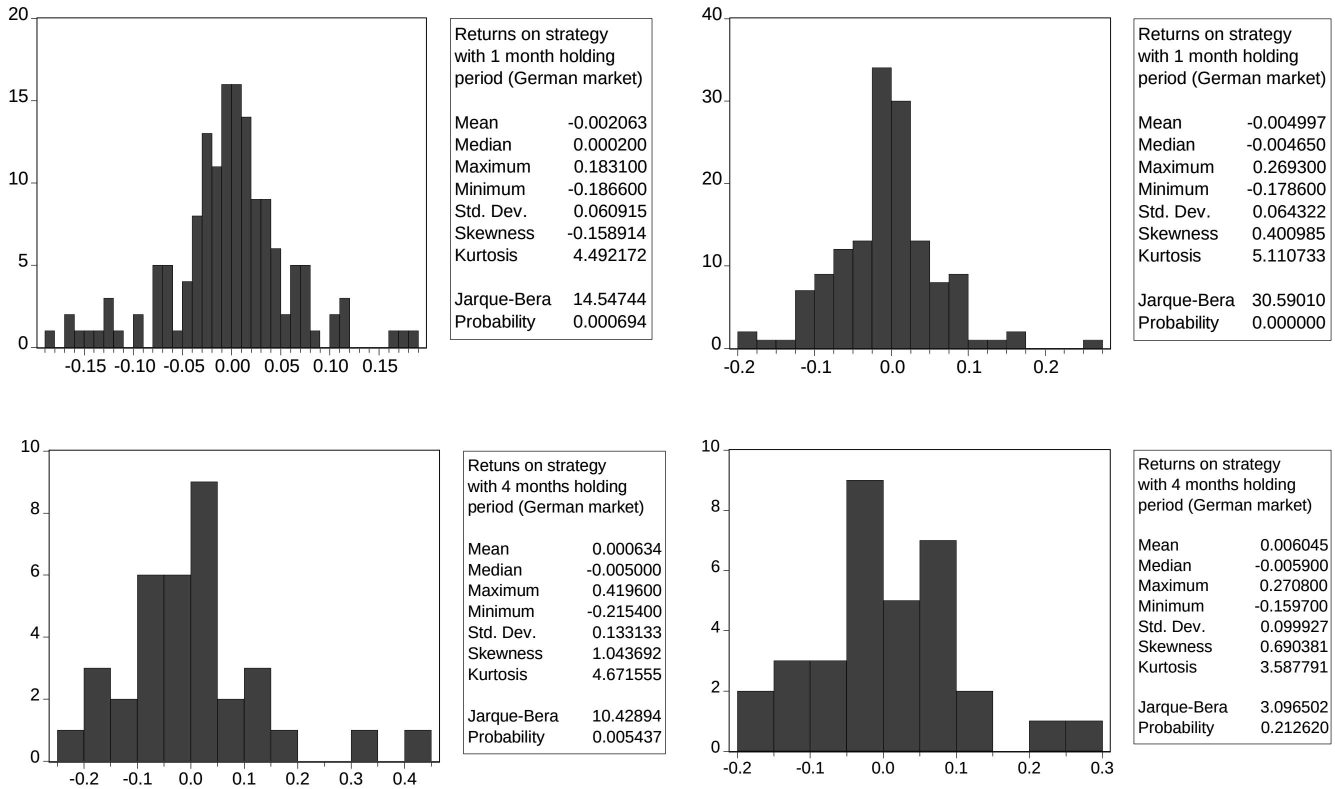

## Table 4.7. Performance Of The French Adjusted Market Index

| Characteristics | 6-month measurement period | | | 1-year measurement period | | |
|---|---|---|---|---|---|---|
| | 1 month | 4 months | 6 months | 1 month | 4 months | 6 months |
| Annualised return | 4.46% | 4.77% | 2.89% | 5.05% | 4.67% | 3.26% |
| Annualised standard deviation | 18.85% | 18.71% | 22.8% | 19.91% | 18.29% | 23.19% |
| Modified Sharpe ratio | 0.2367 | 0.2549 | 0.1268 | 0.2538 | 0.2552 | 0.1407 |
| Skewness | -0.49 | -0.197 | -0.467 | -0.58 | -0.11 | -0.495 |
| Kurtosis | 4.31 | 3.32 | 2.57 | 5.32 | 3.08 | 2.52 |

On French market, our model outperforms the market based on the Sharpe ratio only under the strategies with one-year measurement period. This result was achieved because of the much higher realized return on these strategies whereas the realized volatility of these strategies is higher than that of the market.

## Figure 4.5. Distribution Of Returns On Different Strategies On German Market (Variance Optimisation)

6 months measurement period | 1 year measurement period




## Table 4.8. Performance Of The Variance Minimization Model On German Market

| Characteristics | 6-month measurement period | | | 1-year measurement period | | |
|---|---|---|---|---|---|---|
| | 1 month | 4 months | 6 months | 1 month | 4 months | 6 months |
| Annualised return | -2.63% | 0.19% | -1.04% | -6.37% | 1.8% | 5.77% |
| Annualised standard deviation | 21.75% | 23.1% | 18.04% | 22.97% | 17.31% | 19.01% |
| Beta | -0.2343 | -0.2145 | -0.1427 | -0.4378 | -0.4523 | -0.3646 |
| Modified Sharpe ratio | -0.12 | 0.01 | -0.06 | -0.2774 | 0.105 | 0.03 |
| Skewness | -0.159 | 1.044 | -0.136 | 0.401 | 0.69 | 0.019 |
| Kurtosis | 4.492 | 4.672 | 2.744 | 5.11 | 3.588 | 2.927 |

On German market our model has the worst performance. It has equal number of positive and negative strategies. Therefore, momentum and contrarian strategies are equally likely to be successful. The best performance has the strategy with one-year measurement period and four months holding period. It also has the smallest volatility. But if the contrarian approach were implemented for the strategy with one-year measurement period and one-month holding period, this strategy would have the best performance.

Figure 4.6. Distribution Of German Adjusted Market Index Returns Over Different Periods

6 months measurement period

1 year measurement period

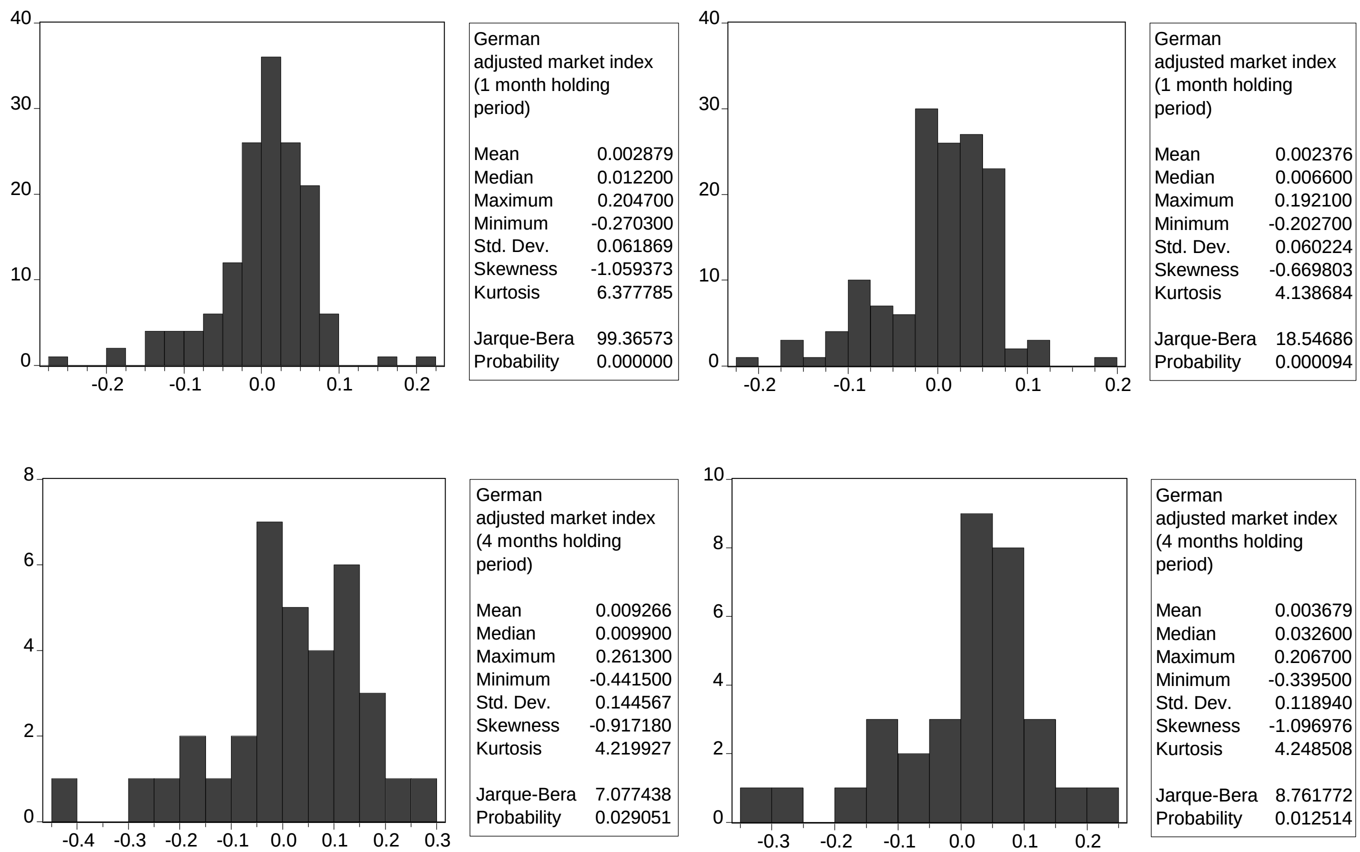


Table 4.9. Performance Of The German Adjusted Market Index

| Characteristics | 6-month measurement period | | | 1-year measurement period | | |
|---|---|---|---|---|---|---|
| | 1 month | 4 months | 6 months | 1 month | 4 months | 6 months |
| Annualised return | 3.67% | 2.78% | 3.6% | 3.03% | 1.104% | 2.75% |
| Annualised standard deviation | 22.09% | 25.04% | 20.52% | 21.5% | 20.6% | 20.77% |
| Modified Sharpe ratio | 0.166 | 0.111 | 0.175 | 0.141 | 0.054 | 0.1323 |
| Skewness | -1.06 | -0.92 | -0.61 | -0.67 | -1.1 | -0.538 |
| Kurtosis | 6.38 | 4.22 | 3.1 | 4.14 | 4.25 | 3.02 |

Most of the time the German adjusted market index outperforms the model. Only one-year estimation period and four-month holding period strategy has twice as high the Sharpe ration as that of the index.

### 4.3.2. Covariance Minimization Under Investment Constraints

$$\max_{W} -|w^{L\prime} cov_{L,S} w^{S}|,$$

$$\text{subject to } \sum_{i=1}^{N+M} w_i = 0,$$

$$\sum_{i=1}^{N} w^{L}{}_{i} = 1,\ \sum_{j=1}^{M} w^{S}{}_{j} = -1,\ 0.1 < w_i^L < 0.6,\ -0.6 < w_i^L < -0.1,$$

where $w^L$ represents portfolio weights of the long position, $w^S$ represents the portfolio weights of the short position, $N$ and $M$ is the number of stocks in the long and short portfolios respectively, and $cov_{L,S}$ is the covariance between the long and the short portfolios. In our case, $N=M=5$. Our goal is to make covariance as low as possible in absolute terms since it can take both positive and negative values.

The optimisation procedure is same as the one described above with the "marginal utilities" for the long and short portfolios as shown below:

$$MU^{long} = \begin{cases} -cov_{L,S}{*}w^{S}, \text{ if } w^{L\prime}cov_{L,S}w^{S}>0, \\ cov_{L,S}{*}w^{S}, \text{ if } w^{L\prime}cov_{L,S}w^{S}<0. \end{cases}$$

$$MU^{short} = \begin{cases} -cov_{L,S'}{*}w^{L}, \text{ if } w^{L\prime}cov_{L,S}w^{S}>0, \\ cov_{L,S'}{*}w^{S}, \text{ if } w^{L\prime}cov_{L,S}w^{S}<0. \end{cases}$$

With

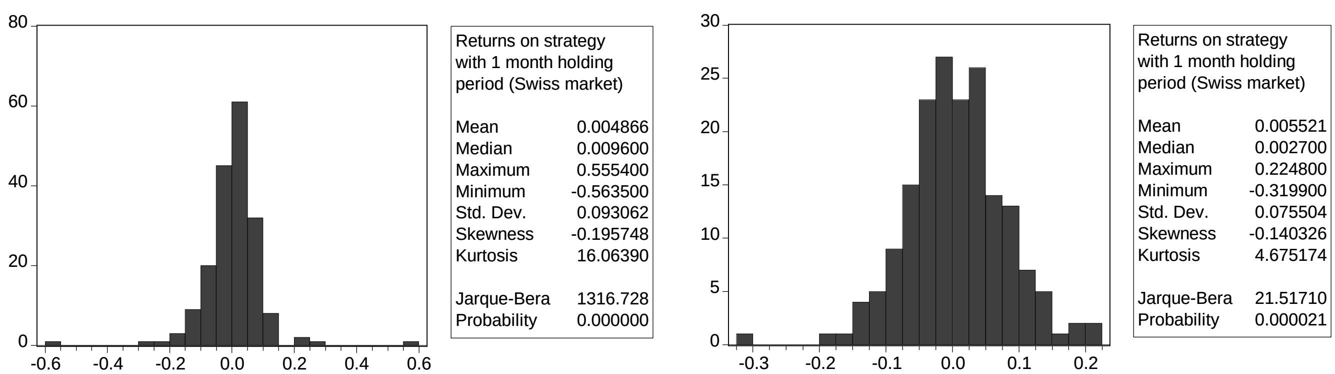
$$c^{Long} = \min\left(-\frac{w^{L}{}'\mathrm{cov}_{L,S}\, s^{S}}{s^{L}{}'\mathrm{cov}_{L,S}\, s^{S}}, w_{U_J} - w_j, w_i - w_i{}^{L}\right),$$

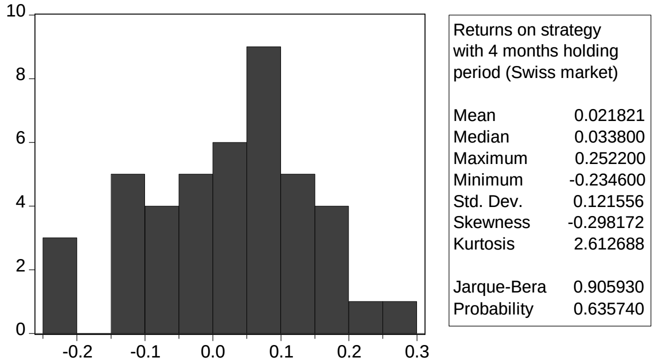
$$c^{Short} = \min\left(-\frac{s^{L}{}'\mathrm{cov}_{L,S}\, w^{S}}{s^{L}{}'\mathrm{cov}_{L,S}\, s^{S}}, w_{U_J} - w_j, w_i - w^{S}{}_{j}\right).$$

On the Swiss, French, and German markets this strategy produces the following results with respect to different holding periods and six-month and one-year estimation periods.

Figure 4.7. Distribution Of Returns On Different Strategies On Swiss Market (Covariance Minimisation)

6 months measurement period 1 year measurement period





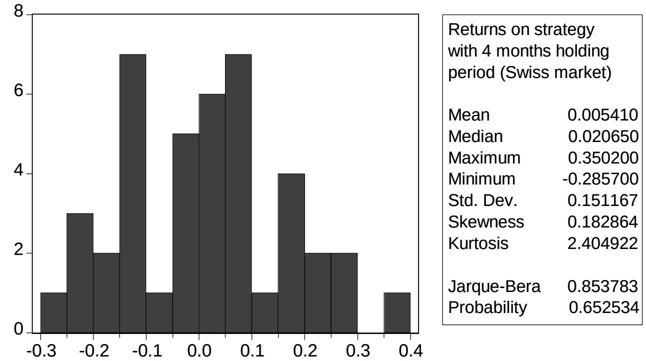



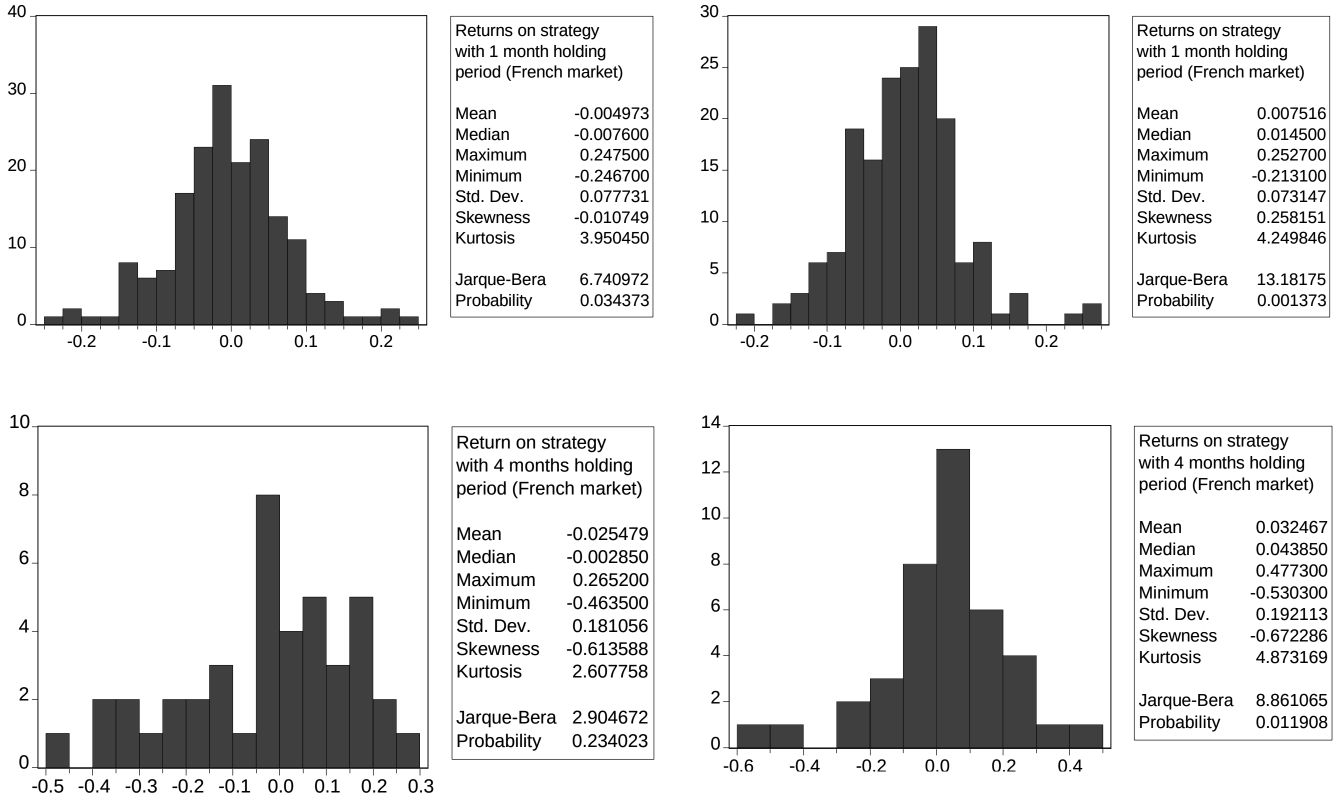

Table 4.10. Performance Of The Covariance Minimization Model On Swiss Market

| Characteristics | 6-month measurement period | | | 1-year measurement period | | |
|---|---|---|---|---|---|---|
| | 1 month | 4 months | 6 months | 1 month | 4 months | 6 months |
| Annualised return | 6.21% | 6.54% | 1.57% | 7.04% | 1.6% | 10.64% |
| Annualised standard deviation | 33.23% | 21.05% | 27.4% | 26.96% | 26.18% | 26.5% |
| Beta | -0.186 | 0.096 | -0.06 | -0.2789 | -0.116 | -0.3582 |
| Modified Sharpe ratio | 0.1867 | 0.3109 | 0.057 | 0.2611 | 0.062 | 0.4016 |
| Skewness | -0.196 | -0.298 | 0.03 | -0.14 | 0.183 | 0.788 |
| Kurtosis | 16.06 | 2.613 | 1.99 | 4.675 | 2.405 | 4.13 |

This model is profitable under all strategies and has the best performance if one year is used as an estimation period and six months are used as a holding period. But since only a part of the total portfolio risk was minimized, this model has higher volatility in all cases than that in the previous models. Comparing with the results of the previous model on the Swiss market, we can conclude that not only the way of stock selection is important but also the optimization approach matters. The realized beta of the model can be acceptable for considering it to be statistical arbitrage only under halve of the strategies, their results are shown in the 3$^{rd}$, 4$^{th}$, and 6$^{th}$ columns. In addition, the performance of the model is much better than that of the Swiss adjusted index besides two strategies with six-month measurement and six-month holding periods and one-year measurement and four-month holding periods.

## Figure 4.8. Distribution Of Returns On Different Strategies On French Market (Covariance minimisation)

6 months measurement period | 1 year measurement period




## Table 4.11. Performance Of The Covariance Minimization Model On French Market

| Characteristics | 6-month measurement period | | | 1-year measurement period | | |
|---|---|---|---|---|---|---|
| | 1 month | 4 months | 6 months | 1 month | 4 months | 6 months |
| Annualised return | -6.34% | -7.64% | -2.08% | 9.58% | 9.74%% | 6.63% |
| Annualised standard deviation | 27.76% | 31.36% | 33.52% | 26.12% | 33.28% | 35.17% |
| Beta | -0.30 | -0.21 | -0.18 | -0.204 | -0.05 | -0.447 |
| Modified Sharpe ratio | -0.228 | -0.2437 | -0.062 | 0.367 | 0.29 | 0.189 |
| Skewness | -0.01 | -0.614 | 0.07 | 0.258 | -0.672 | -0.677 |
| Kurtosis | 3.95 | 2.608 | 3.23 | 4.25 | 4.873 | 3.384 |

Unlike variance minimization approach, this model has three strategies with negative return and three strategies with positive return. The results from the table 4.11 are quite interesting since they show that depending on estimation period either momentum (one-year) or contrarian (six-month) strategy should be implemented to achieve positive return. The realized volatility is higher than in the previous model in all cases. This is not surprising since we only partly minimized total portfolio risk. Under this approach only the strategy with one-month holding period outperforms the corresponding strategy under variance minimization approach, having higher the Sharpe ratio equal to 0.367 versus 0.25. But if contrarian approach were used for six-month estimation period, the covariance approach would have outperformed the variance approach in one-month and six-month holding period strategies.

However, the return in positive-return strategies is almost twice as much as the one generated by the market index over the period these strategies were implemented. This result leads only to higher Sharpe ratios for those strategies.

Figure 4.9. Distribution Of Returns On Different Strategies On German Market (Covariance Minimisation)

6 months measurement period | 1 year measurement period

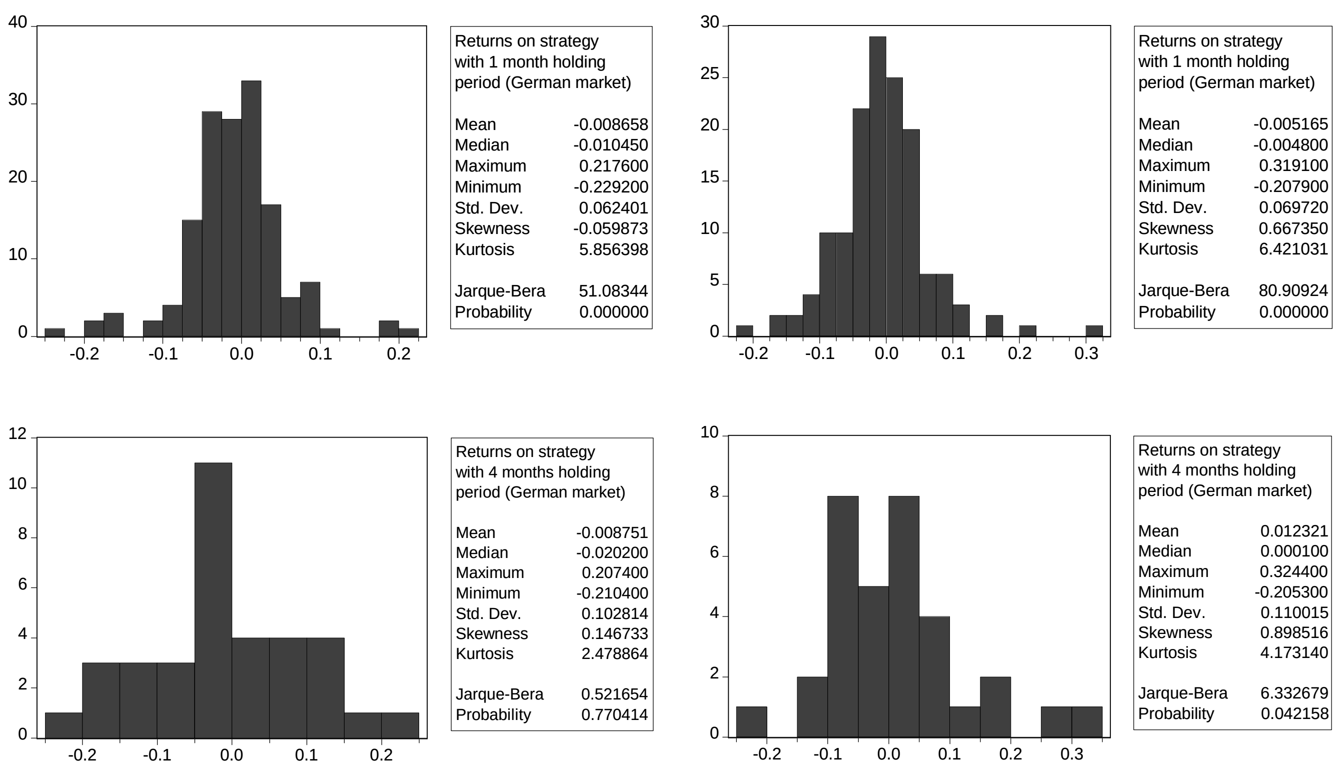


Table 4.12. Performance Of The Covariance Minimization Model On German Market

| Characteristics | 6-month measurement period | | | 1-year measurement period | | |
|---|---|---|---|---|---|---|
| | 1 month | 4 months | 6 months | 1 month | 4 months | 6 months |
| Annualised return | -11.04% | -2.63% | -6.42% | -6.59% | 3.7% | 6.79% |
| Annualised standard deviation | 22.29% | 17.81% | 18.94% | 24.9% | 19.1% | 23.08% |
| Beta | -0.256 | -0.11 | -0.1735 | -0.405 | -0.5 | -0.445 |
| Modified Sharpe ratio | -0.495 | -0.147 | -0.339 | -0.265 | 0.194 | 0.294 |
| Skewness | -0.06 | 0.147 | -0.214 | 0.667 | 0.899 | 0.3112 |
| Kurtosis | 5.86 | 2.479 | 2.524 | 6.42 | 4.17 | 3.102 |

As in the case of variance minimization, the covariance minimization on the German market performs better with contrarian approach, since under momentum we have four out of strategies with negative return. The model has the best result under one-year estimation period and six-month holding period strategy. But this strategy deviates from the statistical arbitrage definition, since its realized beta is equal to –0.445.

### 4.3.3. Optimisation With Zero-Beta

The next portfolio problem that we consider involves market-neutrality condition (zero total portfolio's beta with respect to market). Theoretically, if statistical arbitrage strategy were optimal from investor's point of view, he would look for some optimal tradeoff between portfolio's beta, its risk, and return. The issue is that investor's utility function (expressed in terms of these variables) is unknown. Therefore, we formulate our problem as having an extra constraint of beta strictly equal to zero. The total portfolio risk consists of two parts: systematic and unsystematic risk. By setting zero-beta condition we eliminate the market (systematic) risk in the portfolio, and covariance minimization allows us to eliminate partly unsystematic component. The problem is

$$\max_{w} -|w^{L\prime} cov_{L,S} w^{S}|,$$

$$\text{subject to } \sum_{i=1}^{N+M} w_i = 0, \ \sum_{i=1}^{N+M} w_i b_i = 0, \ \sum_{i=1}^{N+M} w_i R_i = R \ ,$$

$$\sum_{i=1}^{N} w^L{}_i = 1, \ \sum_{j=1}^{M} w^S{}_j = -1, \ 0.1 < w_i^L < 0.6, \ -0.6 < w_i^L < -0.1,$$

Using all these constraints we can express four out of ten portfolio weights using other six weights. First, $w_5$ and $w_{10}$ are found from self-financing constraint.

$$w_5 = 1 - \sum_{i=1}^{N-1} w_i \ , \ w_{10} = 1 - \sum_{j=6}^{N+M-1} w_j \ ,$$

From zero portfolio's beta constraint $w_4$ can be expressed as

$$w_4 = \frac{1}{b_4 - b_5}\left( b_{10} - b_5 + \sum_{j=6}^{9} w_j (b_j - b_{10}) - \sum_{i=1}^{3} w_i (b_i - b_5) \right).$$

Further, from $\sum_{i=1}^{N+M} w_i * R_i = R$ constraint, which we used as equality, $w_3$ is equal to

$$w_3 = \frac{1}{R_4 - R_5}\left( R + R_{10} - R_5 + \sum_{j=6}^{9} w_j (R_j - R_{10}) - \sum_{i=1}^{2} w_i (R_i - R_5) - w_4 (R_4 - R_5) \right).$$

Plugging in the expression for $w_4$ into expression for $w_3$, we obtain

$$w_3 = \left( R_3 - R_5 + \frac{(R_4 - R_5)(b_3 - b_5)}{b_4 - b_5} \right)^{-1} \left\langle \left( aR_M + R_{10} - R_5 + \sum_{j=6}^{9} w_j (R_j - R_{10}) - \sum_{i=1}^{2} w_i (R_i - R_5) \right) - \right.$$
$$\left. - \left( \frac{R_4 - R_5}{b_4 - b_5} \right) \left( b_{10} - b_5 + \sum_{j=6}^{9} w_j (b_j - b_{10}) - \sum_{i=1}^{2} w_i (b_i - b_5) \right) \right\rangle.$$

For the rest of the portfolio allocations we use iteration procedure, which as before considers upper bound at 60% and low bound at 10% level. We take $R$ equal to the return of the portfolio as if it were constructed under variance minimization model. This way we attempt to understand the influence of the market risk component elimination on the performance of the behavioural statistical arbitrage models.

While applying iteration procedure we check whether all the constraints are satisfied. If yes, the program computes and stores the resulted correlation coefficient between long and short parts of the portfolio. When procedure is finished, we pick from the pull of computed covariances the allocation that generates the lowest one.

The described procedure is repeated until the end of the sample is reached. Then, the beta is simply the weighted average of betas of the stocks included in the portfolio

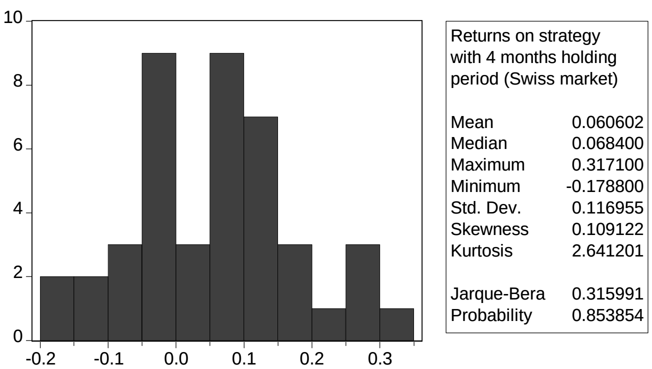
$$b_P = \sum_{j=1}^{N} w_j b_j .$$

Under the same conditions we solved the problem with variance of the whole portfolio being our goal function.

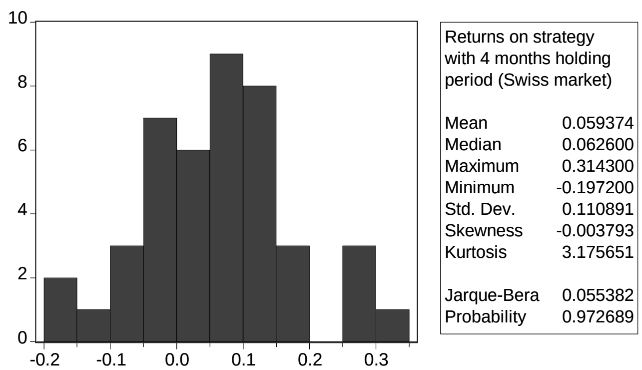
The results of both models are shown below. We tested these models only on the strategies with one-year and six-month estimation periods and four-month holding period, and in iteration procedure we used 5% as a step because of the time constraint.

Figure 4.10. Distribution Of Returns On Different Strategies On Swiss Market (Zero-Beta Strategy)

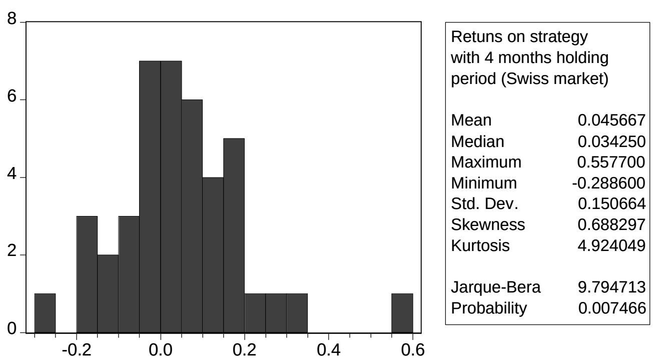
min covariance (6 months measurement period)




min variance (6 months measurement period)




min covariance (1 year measurement period)




min variance (1 year measurement period)

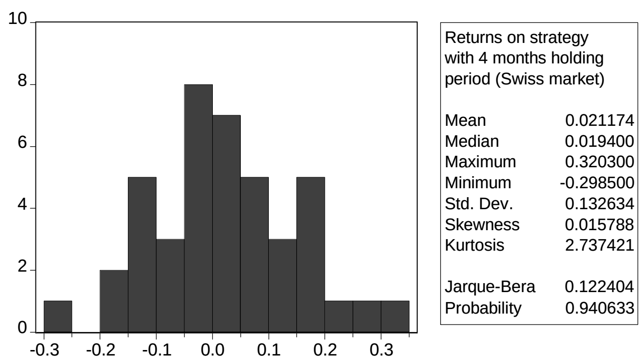


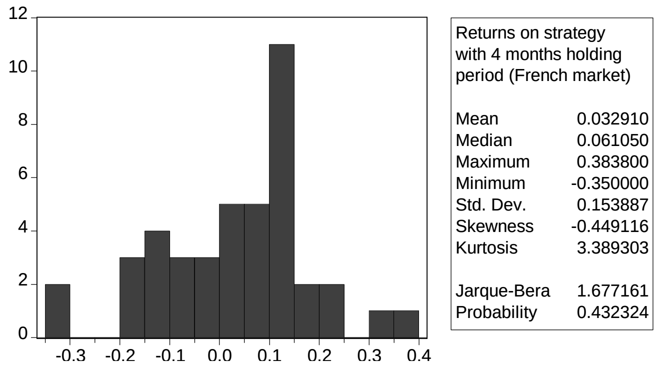
Table 4.13. Performance Of The Zero-Beta Minimization Models On Swiss Market

| Characteristics | 6-month measurement period | | 1-year measurement period | |
|---|---|---|---|---|
| | Min covariance | Min variance | Min covariance | Min variance |
| Annualised return | 18.18% | 17.81% | 13.7% | 6.35% |
| Annualised standard deviation | 20.26% | 19.21% | 26.1% | 22.97% |
| Beta | -0.027 | 0.01 | -0.056 | 0.074 |
| Modified Sharpe ratio | 0.8975 | 0.9274 | 0.525 | 0.277 |
| Skewness | 0.109 | -0.004 | 0.688 | 0.016 |
| Kurtosis | 2.641 | 3.176 | 4.924 | 2.737 |

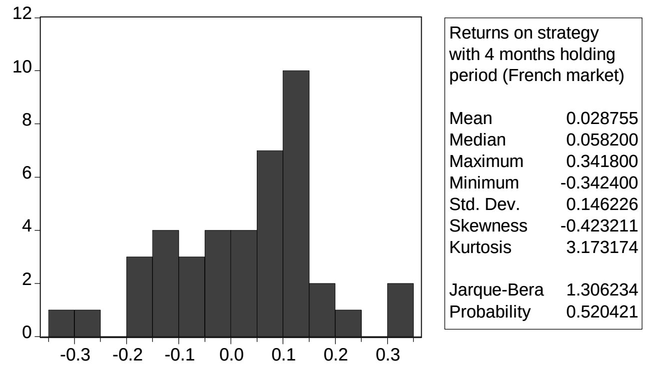
From table 4.13 we can observe that measurement period is crucial for the use of optimisation method. If in case of six-month estimation period the min variance model only slightly outperforms the corresponding strategy under min covariance approach according to Sharpe ratio, then in case of one-year estimation period min covariance that coefficient is twice as big as the one of min variance. However, under six-month estimation period both models have much better performance than under one-year. The same conclusion was made after implementation the first two models on the Swiss market. Since all the constraints, besides zero-beta of the portfolio, were the same, we can deduce from the results that systematic risk component elimination was the key of achieving this better performance. Following Markovitz, we could achieve about the same performance of the first two models, if we had constructed more diversified portfolios (at least 20 stocks). But if small number of stocks is used in the portfolio, market risk reduction is a must to obtain its optimum risk-return profile. The realized beta is much less than in previous models and therefore this model is in line with the required market-neutrality condition.

## Figure 4.11. Distribution Of Returns On Different Strategies On French Market (Zero-Beta Strategy)

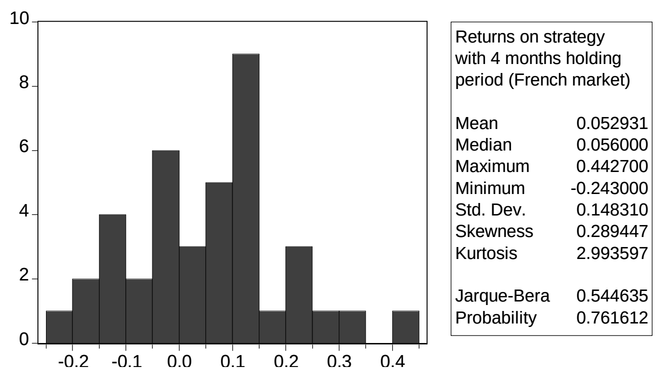
min covariance (6 months measurement period)



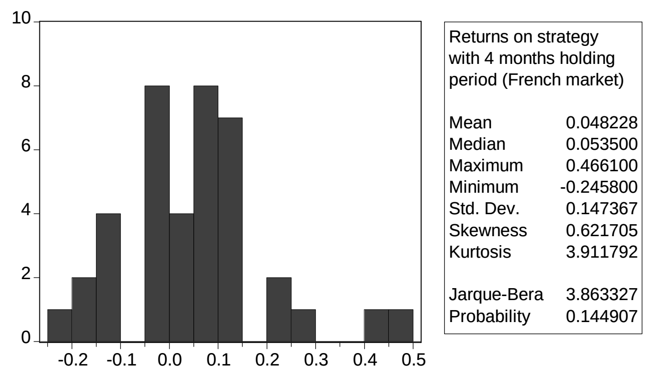

min variance (6 months measurement period)




min covariance (1 year measurement period)




min variance (1 year measurement period)



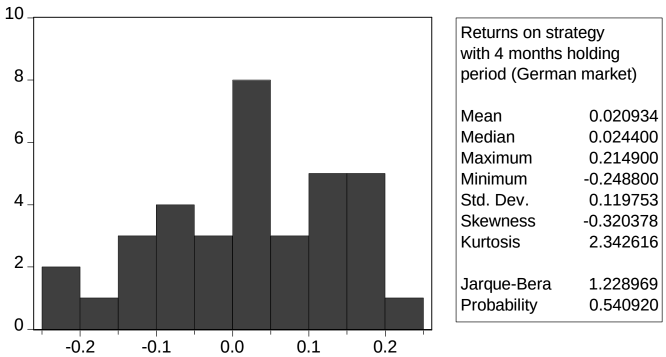

## Table 4.14. Performance Of The Zero-Beta Minimization Models On French Market

| Characteristics | 6-month measurement period | | 1-year measurement period | |
|---|---|---|---|---|
| | Min covariance | Min variance | Min covariance | Min variance |
| Annualised return | 9.87% | 8.63% | 15.66% | 14.47% |
| Annualised standard deviation | 26.65% | 25.33% | 25.69% | 25.53% |
| Beta | -0.017 | -0.049 | -0.226 | -0.008 |
| Modified Sharpe ratio | 0.37 | 0.34 | 0.618 | 0.567 |
| Skewness | -0.449 | -0.423 | 0.289 | 0.622 |
| Kurtosis | 3.389 | 3.173 | 2.994 | 3.91 |

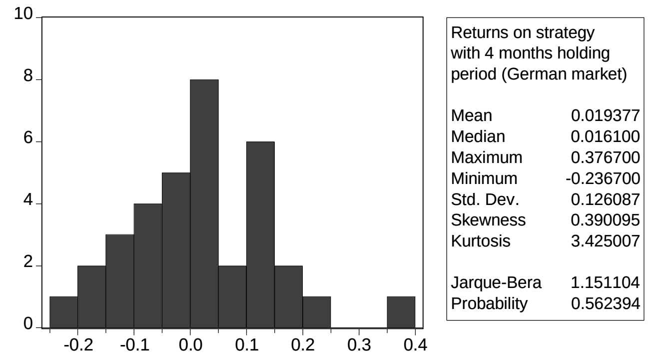
From the table 4.14 we see that min covariance approach performs better that min variance approach under either estimation periods. But as in the first two models, one-year estimation period works better on the French market. The better results were achieved because of systematic risk elimination on the portfolio formation stage. The realized beta of the models stays around zero in almost all cases besides one where min covariance approaches was used under one-year estimation period.

Figure 4.12. Distribution Of Returns On Different Strategies On German Market (Zero-Beta Strategy)

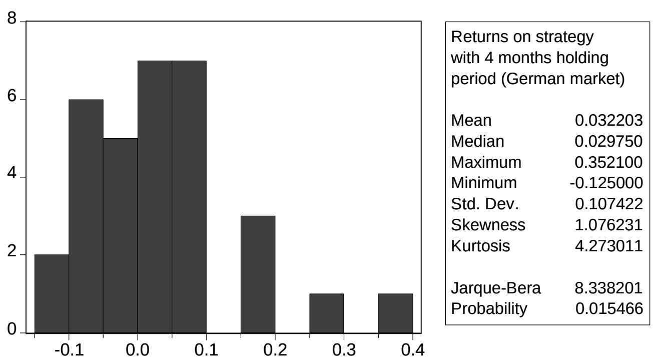
min covariance (6 months measurement period)




min variance (6 months measurement period)




min covariance (1 year measurement period)




min variance (1 year measurement period)

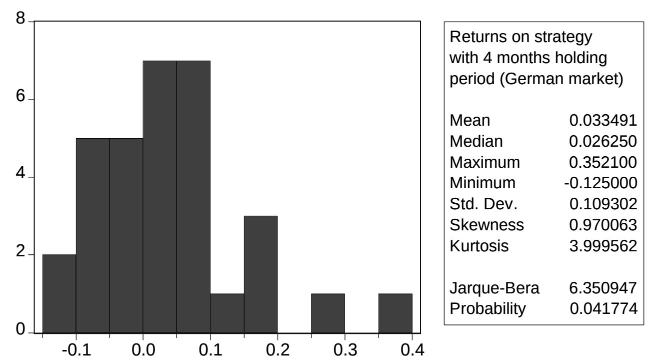



Table 4.15. Performance Of The Zero-Beta Minimization Models On German Market

| Characteristics | 6-month measurement period | | 1-year measurement period | |
|---|---|---|---|---|
| | Min covariance | Min variance | Min covariance | Min variance |
| Annualised return | 6.28% | 5.81% | 9.66% | 10.05% |
| Annualised standard deviation | 20.74% | 21.84% | 18.61% | 18.93% |

| | | | | |
|---|---|---|---|---|
| Beta | -0.1296 | -0.1495 | -0.279 | -0.231 |
| Modified Sharpe ratio | 0.303 | 0.2662 | 0.519 | 0.531 |
| Skewness | -0.32 | 0.39 | 1.076 | 0.97 |
| Kurtosis | 2.34 | 3.43 | 4.273 | 4.0 |

From the above table we can conclude that as on the French market zero-beta models perform better in terms of Sharpe ratio under one-year measurement period for both min variance and min covariance approaches. However, if six-month measurement period is used to select losers and winners, min covariance works better than min variance. If one-year was used, min variance approach has better performance. The realized beta of these last two models is much higher than on the Swiss and French market. However, the realized volatility is smaller. The same is annualised return. The performance of these models is the worst on German market.

### 4.3.4. Comparison With The Price Momentum (Naïve) Strategy

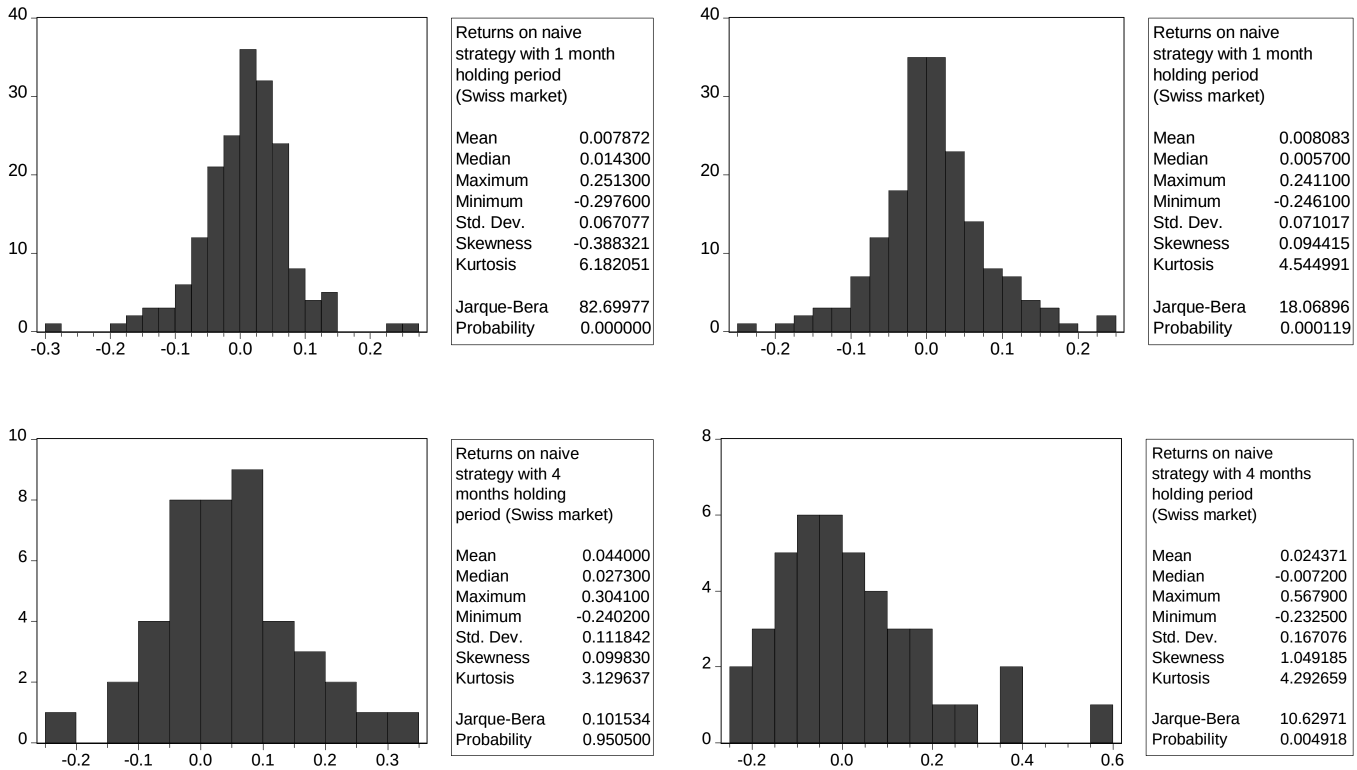
At the end of our empirical research part we would like to compare all our behavioural statistical arbitrage models with the Price Momentum Strategy. We would like to find out how different optimisation approaches affect performance of the momentum strategies.

The simplistic momentum strategy, which is widely used in the academic research papers, represents basically a static portfolio rebalancing after some fixed interval (holding period), ranking based only on the prior price change (over estimation period), and all stocks selected for inclusion in the portfolio have equal weights. In our thesis, we apply the same holding and estimation periods in the momentum strategy as we did in all our models.

The results of the price momentum strategy performance on the Swiss, French, and German markets are shown in the tables 4.16, 4.17, and 4.18 correspondingly.

## Figure 4.13. Distribution Of Returns On Naïve Strategy On Swiss Market

6 months measurement period 1 year measurement period




## Table 4.16. Performance Of The Naïve Strategy On Swiss Market

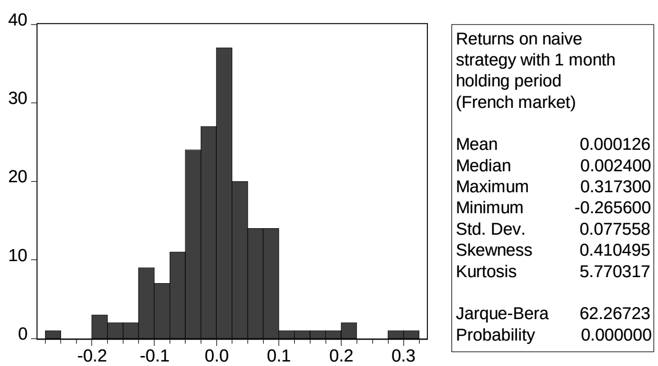
| Characteristics | 6-month measurement period | | | 1-year measurement period | | |
|---|---|---|---|---|---|---|
| | 1 month | 4 months | 6 months | 1 month | 4 months | 6 months |
| Annualised return | 10.04% | 13.2% | 9.94% | 10.31% | 7.31% | 11.87% |
| Annualised standard deviation | 23.95% | 19.37% | 22.31% | 25.36% | 28.94% | 22.16% |
| Beta | -0.328 | -0.1168 | -0.1526 | -0.396 | -0.310 | -0.24 |
| Modified Sharpe ratio | 0.419 | 0.68 | 0.4455 | 0.406 | 0.2527 | 0.5359 |
| Skewness | -0.388 | 0.1 | -0.05 | 0.09 | 1.05 | -0.115 |
| Kurtosis | 6.18 | 3.13 | 2.07 | 4.54 | 4.29 | 2.457 |

Looking at the results of the naïve strategy presented in the table 4.16 and comparing them with the results of our models shown in the tables 4.4, 4.10, and 4.13, we can conclude that the naïve strategy outperforms min variance and min covariance approaches based on the Sharpe ratio. There is only one exception. Under min variance model the strategy with one-year measurement period and one-month holding period slightly outperforms the Sharpe measure because of the smaller volatility. Overall, volatility of the min variance approach in smaller than that of naïve strategy, and this is not surprising. However, volatility of the min covariance approach is higher and in some strategies is much higher than volatility of the naïve strategy. This happened because on the stage of portfolio formation we have not controlled the total risk of the portfolio, but only a pert of it. The bet that independent movements in the long and short parts of the total portfolio would lead to much higher return than the return of the min variance approach. Unfortunately, it did not work.

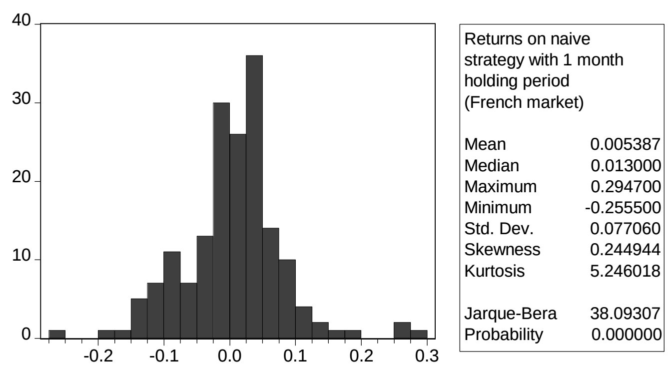
The zero-bate approach is the only one that shows much better results than the naïve strategy. It has higher return and smaller volatility. This fact represents necessity of market risk minimization on the portfolio formation stage. Therefore, inclusion of bigger number of stocks in the portfolio will be a solution to better performance of the min variance and min covariance models. We did not test zero-beta approach on all of the strategy, but we can expect that it will also outperform the price momentum strategy.

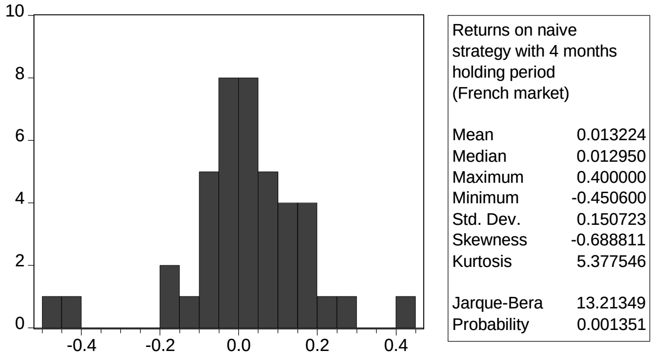
Figure 4.14. Distribution Of Returns On Naïve Strategy On French Market

6 months measurement period




1 year measurement period





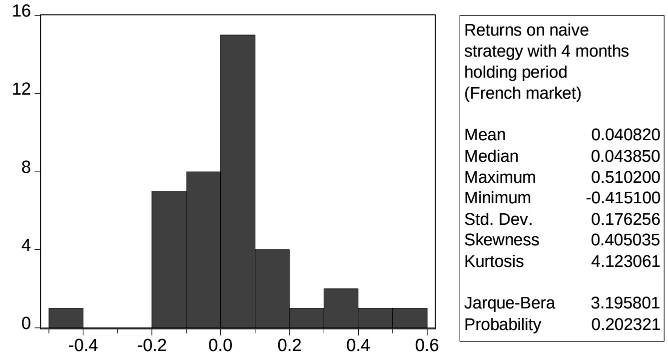




Table 4.17. Performance Of The Naïve Strategy On French Market

| Characteristics | 6-month measurement period | | | 1-year measurement period | | |
|---|---|---|---|---|---|---|
| | 1 month | 4 months | 6 months | 1 month | 4 months | 6 months |
| Annualised return | 0.16% | 3.97% | 2.97% | 6.87% | 12.25% | 11.31% |
| Annualised standard deviation | 27.7% | 26.11% | 30.52% | 27.52% | 30.5% | 34.87% |
| Beta | -0.502 | 0.01 | -0.2354 | -0.36 | -0.277 | -0.5891 |
| Modified Sharpe ratio | 0.006 | 0.152 | 0.1 | 0.2496 | 0.40 | 0.3243 |
| Skewness | 0.41 | -0.69 | 0.15 | 0.24 | 0.405 | 0.53 |
| Kurtosis | 5.77 | 5.38 | 3.81 | 5.25 | 4.12 | 4.497 |

Comparing results on the naïve strategy on the French market shown in the table 4.17 with results of our models presented in the tables 4.6, 4.11, and 4.14, we can conclude that as on the Swiss market, the price momentum strategy has better performance than min variance and min covariance approaches, and does not have any strategies with negative return. However, volatility of min variance approach is less, but volatility the of min covariance approach is slightly higher. The reason is explained above.

Zero-beta approach on the French market also beats the price momentum as on the Swiss market. It has higher realized return and smaller realized volatility. We would also expect this approach to outperform the naïve strategy in other cases of measurement and holding periods.

Figure 4.15. Distribution Of Returns On Naïve Strategy On German Market

6 months measurement period 1 year measurement period

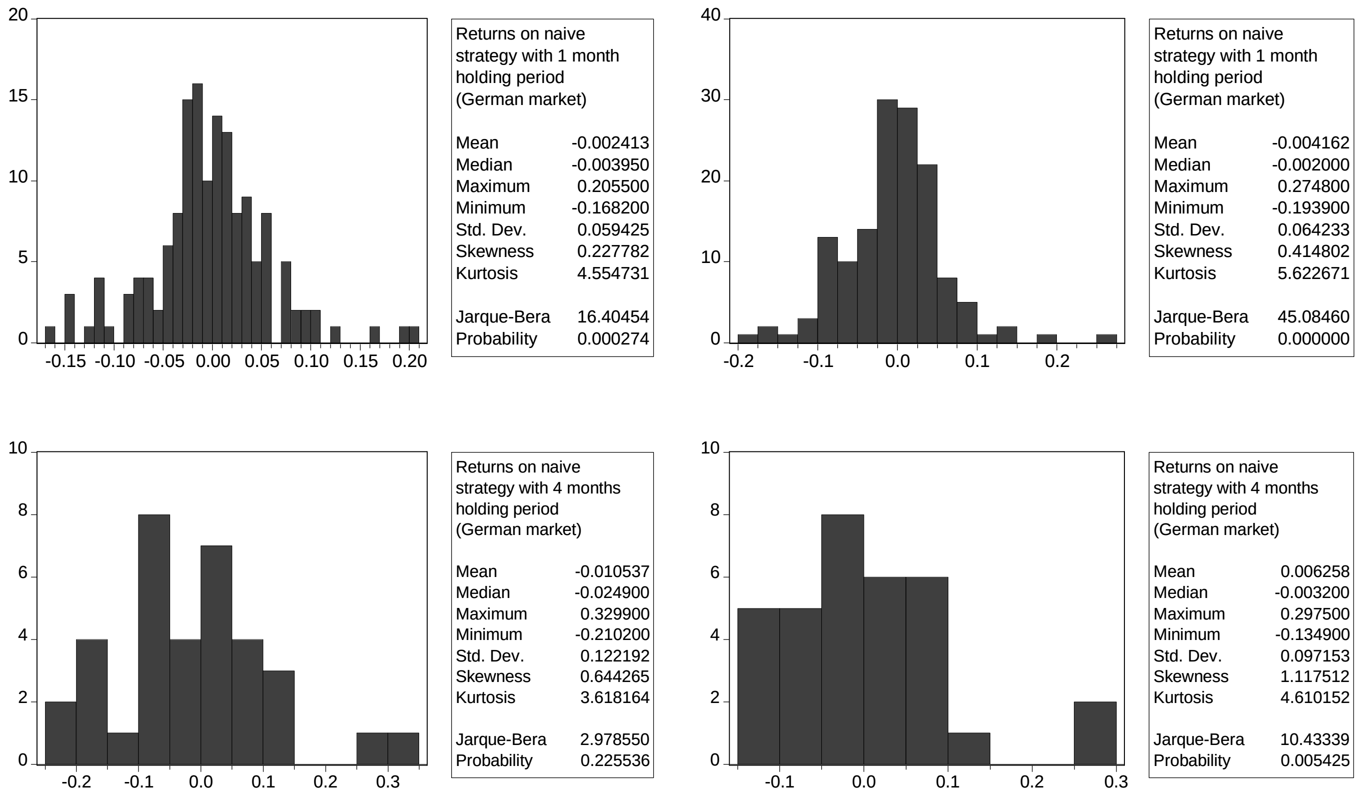



Table 4.18. Performance Of The Naïve Strategy On German Market

| Characteristics | 6-month measurement period | | | 1-year measurement period | | |
|---|---|---|---|---|---|---|
| | 1 month | 4 months | 6 months | 1 month | 4 months | 6 months |
| Annualised return | -3.08% | -3.16% | -1.45% | -5.52% | 1.88% | 1.95% |
| Annualised standard deviation | 21.22% | 21.16% | 16.68% | 22.87% | 16.83% | 18.86% |
| Beta | -0.346 | -0.24 | -0.1764 | -0.4622 | -0.475 | -0.361 |
| Modified Sharpe ratio | -0.145 | -0.149 | -0.087 | -0.2412 | 0.112 | 0.1035 |
| Skewness | 0.228 | 0.64 | -0.063 | 0.415 | 1.12 | 0.216 |
| Kurtosis | 4.55 | 3.62 | 2.99 | 5.62 | 4.61 | 3.142 |

Performance of the price momentum on the German market is the worst. The results are shown in the table 4.18. It has four out of six strategies with negative returns whereas on the Swiss and French markets it has none. Comparing these results with the results of the min variance, min covariance, and zero-beta approaches that are presented in the tables 4.8, 4.12, and 4.15 respectively, we can conclude that in those periods where price momentum has positive returns it outperforms two first models having smaller volatility because of the smaller realized volatility. Only in one case min variance approach outperforms the price momentum. It happened when six-month measurement and four-month holding period were used. This strategy has 0.19% realized return whereas the price momentum has it –3.16%. However, if we used contrarian instead of momentum methodology in those cases when the price momentum has negative returns, we would have much better performance of the min covariance model.

The results of the last (zero-beta) model on the German market are consistent with results of this model on the Swiss and French markets. These results were achieved because of much higher realized return, since realized volatility in most cases is slightly higher than that of the price momentum strategy.

# 5. Conclusions

Below we summarise the results that we have obtained during the simulations under different investment constraints.

As it was explained above, in our behavioural statistical arbitrage strategy we used three portfolio optimisation models:

1. Variance minimisation
2. Covariance minimisation
3. Minimization of portfolio variance and covariance between long and short portfolios under zero-beta condition.

Our results prove, that it is possible to outperform the market using behavioural statistical arbitrage strategy and portfolio optimisation techniques explained above. The best results are observed on the Swiss market, where the degree of outperformance of the strategy comparing to index is the largest. Then follows French market and the lowest degree of outperformance of the strategy is observed on the German market.

The best results our strategy generated for the Swiss market, where the number of successful outcomes is the largest and the best measurement and holding periods are the same as for the classical momentum – 6 months and 4 months respectively. For French and German markets the best measurement period is longer than for the Swiss market and is equal to one year, while holding period is 4 months. Our strategy gives the worst results for the German market with the smallest number of positive outcomes.

For all markets the portfolio optimisation technique, which generates the best results is the zero-beta minimisation strategy. As above, the best result for Swiss market is generated on 6-month measurement period, while for French and German markets the best measurement period is again 1 year. For Swiss and German markets we got the best results when we minimised the portfolio variance, while for French market – when we minimise the covariance between long and short positions.

Thus, we can make a conclusion, that there is no common model that can be applied on all of the chosen markets. This can be explained by national specifics of the markets, number of active participants on the markets and stocks available.

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