A regime-switching relative value arbitrage rule

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1 Introduction

The relative value arbitrage rule, also known as "pairs trading" or "statistical arbitrage", is a well established speculative investment strategy on financial markets, dating back to the 1980s. Today, especially hedge funds and investment banks extensively implement pairs trading as a long/short investment strategy.[1]

Based on relative mispricing between a pair of stocks, pairs trading strategies create excess returns if the spread between two normally comoving stocks is away from its equilibrium path and is assumed to be mean reverting, i.e. deviations from the long term spread are only temporary effects. In this situation, pairs trading suggests to take a long position in the relative undervalued stock, while the relative overvalued stock should be shortened. The formation of the pairs ensues from a cointegration analysis of the historical prices. Consequently, pairs trading represents a form of statistical arbitrage where econometric time series models are applied to identify trading signals.

However, fundamental economic reasons might cause simple pairs trading signals to be wrong. Think of a situation in which a profit warning of one of the two stocks entails the persistant widening of the spread, whereas for the other no new information is circulated. Under these circumstances, betting on the spread to revert to its historical mean would imply a loss.

To overcome this problem of detecting temporary in contrast to longer lasting deviations from spread equilibrium, this paper bridges the literature on Markov regime-switching and the scientific work on

[1] For an overview see [7, 3].

statistical arbitrage to develop useful trading rules for "pairs trading". The next section contains a brief overview of relative value strategies. Section 3 presents a discussion of Markov regime-switching models which are applied in this study to identify pairs trading signals (section 4). Section 5 presents some preliminary empirical results for pairs of stocks being derived from DJ STOXX 600. Section 6 concludes with some remarks on potential further research.

2 Foundations of relative value strategies

Empirical results, documented in the scientific literature on relative value strategies, indicate that the price ratio $Rat_t = \left(P_t^A / P_t^B\right)$ of two assets A and B can be assumed to follow a mean reverting process [3, 7]. This implies that short term deviations from the equilibrium ratio are balanced after a period of adjustment. If this assumption is met, the "simple" question in pairs trading strategies is that of discovering the instant where the spread reaches its maximum and starts to converge.

The simplest way of detecting these trading points is to assume an extremum in Rat_t when the spread deviates from the long term mean by a fixed percentage. In other cases confidence intervals of the ratio's mean are used for the identification of trading signals.[2] Higher sophisticated relative value arbitrage trading rules based on a Kalman filter approach are provided in [2, 1].

Pairs trading strategies can be divided into two categories in regard to the point in time when a trade position is unwinded. According to conservative trading rules the position is closed when the spread reverts to the long term mean. However, in risky approaches the assets are held until a "new" minimum or maximum is detected by the applied trading rule.

However, one major problem in pairs trading strategy - besides the successful selection of the pairs - stems from the assumption of mean reversion of the spread. Pairs traders report that the mean of the price ratio seems to switch between different levels and traditional technical trading approaches often fail to identify profit opportunities. In order to overcome this problem of temporary vs. persistent spread deviations, we apply a Markov regime-switching model with switching mean and switching variances to detect such phases of imbalances.

[2] See [3].

3 Markov regime-switching model

Many financial and macroeconomic time series are subject to sudden structural breaks [5]. Therefore, Markov regime-switching models have become very popular since the late 1980s. In his seminal paper Hamilton [4] assumes that the regime shifts are governed by a Markov chain. As a result the current regime s_t is determined by an unobservable, i.e. latent variable. Thus, the inference of the predominant regime is based only on calculated state probabilities. In the majority of cases a two-state, first-order Markov-switching process for s_t is considered with the following transition probabilities [6]:

$$prob\left[s_t = 1 | s_{t-1} = 1\right] = p = \frac{\exp\left(p_0\right)}{1 + \exp\left(p_0\right)} \tag{1}$$

$$prob\left[s_t = 2 | s_{t-1} = 2\right] = q = \frac{\exp\left(q_0\right)}{1 + \exp\left(q_0\right)}, \tag{2}$$

where p_0 and q_0 denote unconstrained parameters. We apply the following simple regime-switching model with switching mean and switching variance for our trading rule:

$$Rat_t = \mu_{s_t} + \varepsilon_t, \tag{3}$$

where $E\left[\varepsilon_t\right] = 0$ and $\sigma^2_{\varepsilon_t} = \sigma^2_{s_t}$.

To visualize the problem of switching means figure 1 plots a time series of a scaled price ratio, where the two different regimes are marked. The shaded area indicates a regime with a higher mean (μ_{s_1}) while the non-shaded area points out a low-mean regime (μ_{s_2}).

Traditional pairs trading signals around the break point BP would suggest an increase in Rat_{BP} implying a long position in Anglo American PLC and a short position in XSTRATA PLC. As can be seen in figure 1 this trading position leads to a loss, since the price of the second stock relative to the price of the first stock increases.

4 Regime-switching relative value arbitrage rule

In this study we suggest applying Markov regime-switching models to detect profitable pairs trading rules. In a first step we estimate the Markov regime-switching model as stated in equation (3). As a byproduct of the Markov regime-switching estimation we get the smoothed probabilities $P\left(\cdot\right)$ for each state. Based on these calculated probabilities we identify the currently predominant regime. We assume a two-state process for the spread and interpret the two regimes as a *low* and

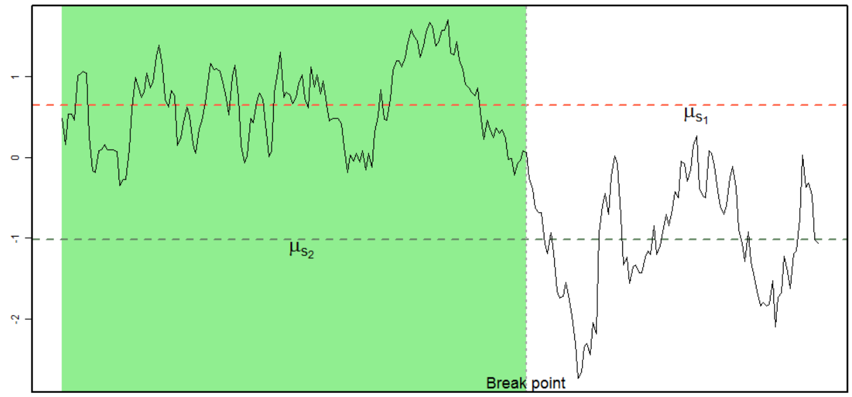

Fig. 1. Scaled ratio of the stock prices of Anglo American PLC and XSTRATA PLC from 2006-12-01 to 2007-11-15. The ratio exhibits a switching mean. The shaded area indicates the high mean regime.

a *high* mean regime. In consequence, we try to detect the instant where the spread Rat_t reaches a local extremum. As a matter of convenience, we adopt the traditional pairs trading approach that a minimum or maximum is found when the spread deviates from the mean by a certain amount. However, we extend the traditional rule by considering a low and a high mean regime, and so we create a regime dependent arbitrage rule. A trading signal z_t is created in the following way:

$$z_t = \begin{cases} -1 \quad if \quad Rat_t \geq \mu_{s_t} + \delta \cdot \sigma_{s_t} \\ +1 \quad if \quad Rat_t \leq \mu_{s_t} - \delta \cdot \sigma_{s_t}, \end{cases} \tag{4}$$

otherwise $z_t = 0$. We use δ as a standard deviation sensitivity parameter and set it equal to 1.645. As a result, a local extremum is detected, if the current value of the spread lies outside the 90% confidence interval within the prevailing regime. The interpretation of the trading signal is quite simple: if $z_t = -1$ $(+1)$ we assume that the observed price ratio Rat_t has reached a local maximum (minimum) implying a short (long) position in asset A and a long (short) position in asset B.

Probability threshold

To evaluate the trading rule dependent on the current regime (*low* or *high* mean), we additionally implement a probability threshold ρ in our arbitrage rule. Therefore, the regime switching relative value arbitrage rule changes in the following way:

$$z_t = \begin{cases} -1 & if \quad Rat_t \geq \mu_{low} + \delta \cdot \sigma_{low} \wedge P(s_t = low|Rat_t) \geq \rho \\ +1 & if \quad Rat_t \leq \mu_{low} - \delta \cdot \sigma_{low} \end{cases} \tag{5}$$

otherwise $z_t = 0$, if s_t is in the *low mean regime*. In the *high mean regime* a trading signal is created by:

$$z_t = \begin{cases} -1 & if \quad Rat_t \geq \mu_{high} + \delta \cdot \sigma_{high} \\ +1 & if \quad Rat_t \leq \mu_{high} - \delta \cdot \sigma_{high} \wedge P(s_t = high|Rat_t) \geq \rho \end{cases} \tag{6}$$

otherwise $z_t = 0$. The probabilities $P(\cdot)$ of each regime indicate whether a structural break is likely to occur. If the probability suddenly drops from a high to a lower level, our regime switching relative value arbitrage rule prevents us from changing the trading positions the wrong way around, so that a minimum or a maximum is not detected too early. The probability threshold value is set arbitrarily. Empirical results suggest a setting for ρ ranging from 0.6 to 0.7. Therefore, the trading rule acts more cautiously in phases where the regimes are not selective.

5 Empirical results

The developed investment strategy is applied in a first data set to the investing universe of the DJ STOXX 600. Our investigation covers the period 2006-06-12 to 2007-11-16. We use the first 250 trading days to find appropriate pairs, where we use a specification of the ADF-test for the pairs selection. The selected pairs[3] are kept constant over a period of 50, 75, 100 and 125 days. However, if a pair sustains a certain accumulated loss (10%, 15%), it will be stopped out. To estimate the parameters of the Markov regime-switching model we use a rolling estimation window of 250 observations.

For reasons of space, only one representative example will be quoted. Table 1 demonstrates the results of the regime-switching relative value arbitrage rule for the second term of 2007. In this period the best result (average profit of 10.6% p.a.) is achieved by keeping the pairs constant over 125 days and by a stop loss parameter of 15%. The setting of 50 days with a stop loss of 10% generates an average loss of -1.5% p.a. It should be noted that the trading and lending costs (for short selling) have not been considered in this stage of the study.

[3] A number of 25 was detected. One asset is only allowed to occur in 10% of all pairs because of risk management thoughts.

panel	50 days		75 days		100 days		125 days	
stop loss	10%	15%	10%	15%	10%	15%	10%	15%
μ	-0.01470	0.00555	0.05022	0.07691	0.03038	0.05544	0.08196	0.10617
σ	0.17010	0.17568	0.18181	0.19042	0.19105	0.20417	0.21265	0.23059
min	-0.40951	-0.40951	-0.29616	-0.38670	-0.23157	-0.23402	-0.19000	-0.22644
1Q	-0.21938	-0.18922	-0.15507	-0.08395	-0.23157	-0.10718	-0.19000	-0.19000
2Q	0.00000	0.00823	0.00000	0.09495	-0.02199	0.05935	0.06252	0.06785
3Q	0.18277	0.18277	0.21685	0.21685	0.18718	0.18718	0.23587	0.23587
max	0.79171	0.79171	0.84898	0.84898	0.60407	0.60407	1.27242	1.27242
#	23	24	21	24	18	23	15	19

Table 1. Annualized descriptive statistics for the over all selected pairs averaged results of the second term of 2007. # denotes the number of pairs not leading to a stop loss.

6 Conclusion

In this study we implemented a Markov regime-switching approach into a statistical arbitrage trading rule. As a result a regime-switching relative value arbitrage rule was presented in detail. Additionally, the trading rule was applied for the investing universe of the DJ STOXX 600. The empirical results, which still remain to be validated, suggest that the regime-switching rule for pairs trading generates positive returns and so it offers an interesting analytical alternative to traditional pairs trading rules.

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